Comparison of sensitivity computations

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1 Objective

This technical note compares the sensitivity curves obtained by the ASWG from the Monte Carlo data to the sensitivity curves obtained from the Instrument Response Function (IRF) FITS files and the sensitivity estimate obtained using the `cssens` script. Sensitivity curves from the ASWG have been extracted from the latest Prod3b ROOT response files. Here only the response of the Southern baseline array is considered for a zenith angle of 20°. The IRF FITS files have been generated from the ROOT file using the `csroot2caldb` script. The ctools version 1.3.0.dev1 from 6 April 2017 has been used for the comparison.

2 Sensitivity computations

2.1 Computation of sensitivity from the FITS IRF using the Li & Ma formula

The detection sensitivity can be computed from the Instrument Response Function FITS files using the formula of Li & Ma [1]. As first step, the number of background counts $N_{\rm bgd}$ within a given energy interval

$$[E_{\rm min}, E_{\rm max}] = [10^{\log E_0 - 0.1}, 10^{\log E_0 + 0.1}] \tag{1}$$

is computed, where E_0 is the logarithmic mean of the energy interval in TeV. The computation is performed using

$$N_{\rm bgd} = T \times \Omega \times \int_{E_{\rm min}}^{E_{\rm max}} B(E) dE \tag{2}$$

where the background rate $B(E)$, given in units of $\rm s^{-1} sr^{-1} MeV^{-1}$, is integrated over the energy interval $[E_{\rm min}, E_{\rm max}]$, T is the observing time in seconds, and Ω is the solid angle subtended by the angular cut in steradians. We employ here an angular cut that corresponds to the 68% containment radius of the Point Spread Function (PSF). The number of "off" counts is then derived using

$$N_{\rm off} = N_{\rm bgd}/\alpha \tag{3}$$

where $\alpha = 0.2$, which corresponds to an "off" region that is five times larger than the "on" region.

From the number of "off" counts the number of "on" counts is inferred by iteratively solving Equation (17) of Li & Ma [1]

$$S = \sqrt{2} \left\{ N_{\rm on} \ln \left[\frac{1+\alpha}{\alpha} \left(\frac{N_{\rm on}}{N_{\rm on} + N_{\rm off}} \right) \right] + N_{\rm off} \ln \left[(1+\alpha) \left(\frac{N_{\rm off}}{N_{\rm on} + N_{\rm off}} \right) \right] \right\}^{1/2} \tag{4}$$

for $N_{\rm on}$ using $S = 5$. From that, the number of source counts within the 68% PSF containment radius is computed using

$$N_{\rm src} = N_{\rm on} - N_{\rm bgd} \tag{5}$$

The sensitivity in units of erg cm^{-2} s^{-1} is then computed using

$$E^2 dN/dE = 1.6022 \times E^2 \frac{N_{\rm src}}{N_{\rm pred}(E)} \tag{6}$$

where

$$N_{\rm pred}(E) = 0.68 \times T \times \int_{E_{\rm min}}^{E_{\rm max}} A(E)\,(E/E_0)^{-2.48}\,dE \tag{7}$$

is the expected number of source counts for a source with a power law spectrum with spectral index of -2.48 and a pre factor of 1 ph cm^{-2} s^{-1} TeV^{-1}. The exact value of the spectral index is not crucial as long as the energy interval is sufficiently small. T is the observing time in seconds and the factor 0.68 takes into account that only 68% of the events are falling within the "on" region. The factor $1.6022 \times E^2$ converts from TeV^{-1} to erg.

In a variant of the method the constraints

$$N_{\rm src} \geq 0.05 \times N_{\rm bgd} \tag{8}$$

and

$$N_{\rm src} \geq 10 \tag{9}$$

are added. The first constraint limits the sensitivity at low energies, and takes into account that the sensitivity is limited by background systematics. The second constraint limits the sensitivity at high energies, and requires at least source 10 photons.

2.2 cssens

The `cssens` script computes for a number of energy bins the flux of a source that is detected with a Test Statistic value of 25 corresponding to a significance of 5σ. Starting from an initial source flux of 100 mCrab, the `cssens` script simulates an observation, fits the test source to the simulated data, and determines its Test Statistic value. Depending on whether the Test Statistic is larger or smaller than the target value of 25, the flux of the test source is reduced or increased, and the procedure repeated. By adjusting a regression curve to the pairs of flux and Test Statistic values, `cssens` estimates the flux that corresponds to a Test Statistic value of 25. Once the estimated flux stabilises, the iterative procedure terminates. The sensitivity is then computed using

$$E^2 dN/dE = 1.6022 \times E_0^2 \times F(E_0) \tag{10}$$

where E_0 is the logarithmic mean energy of the energy bin, $F(E_0)$ is the spectral flux density of the stabilised test source, and the factor 1.6022 assures that the result is in units of erg cm^{-2} s^{-1}.

3 Comparison

Figures 1 - 3 compare the different sensitivity computations. The green curves show the sensitivities as extracted from the ROOT files. Dark green corresponds to the on-axis sensitivity while light green is extracted from the 2-dimensional offaxis response histograms. Apparently there are only very minor differences between the histograms.

The blue curves show the sensitivities computed from the FITS IRFs using the Li & Ma formula (see section 2.1). The dark blue curves correspond to the computation without any constraints, the light blue curves correspond to the computation with constraints. Introducing the constraints (Equations 8 - 9) on the number of source photons only slightly degrades the sensitivity (difference between dark blue and light blue).

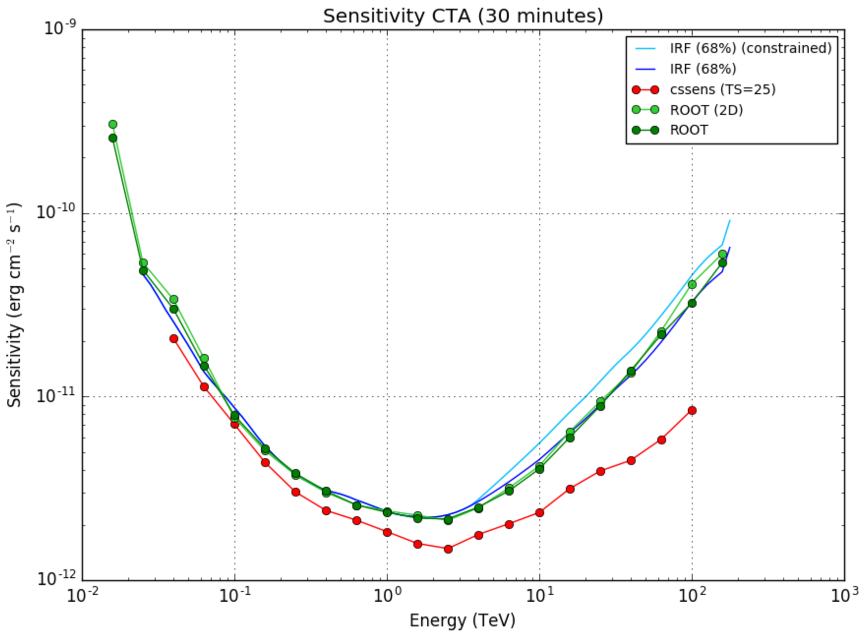

Figure 1: Comparison of sensitivities for 30 minute cuts.

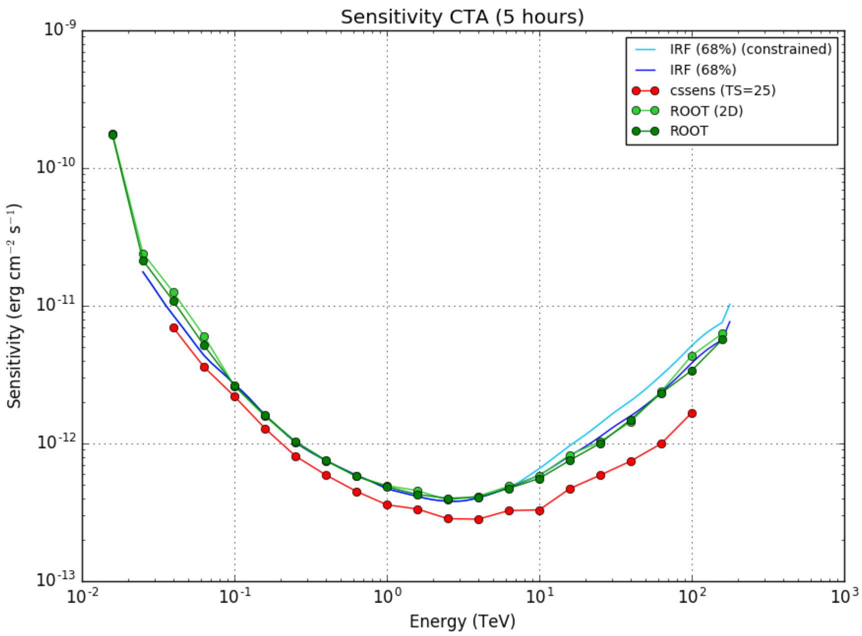

Figure 2: Comparison of sensitivities for 5 hours cuts.

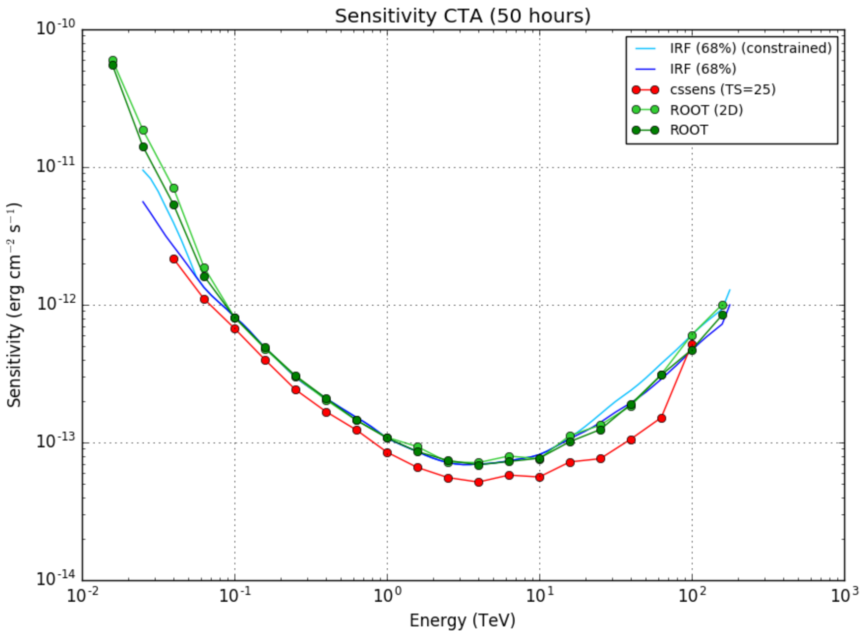

Figure 3: Comparison of sensitivities for 50 hours cuts.

The red curves correspond to the results obtained using `cssens`. In all cases the sensitivity predicted by `cssens` is superior to the sensitivities predicted by the other methods. While the other methods determine the sensitivity using traditional analysis methods that apply an angular cut (θ^2-cut) and examine the counts in "on" and "off" regions, `cssens` uses the full data space without applying any angular cut in the analysis, taking into account the full spatial and spectral information that is available. `cssens` imposes no constraint on the minimum number of detected source photons which naturally leads to better sensitivities at high energies.

Fitting the data over the full data space also leads to better constraints of the background estimation. This is illustrated in Figure 4 where α has been set to 10^{-4} for the IRF sensitivities to emulate a better statistics for the background determination. Now, the dark blue sensitivity curve is close to the `cssens` sensitivity curve, and the $N_{\rm src} \geq 10$ constraint has a strong impact at high energies on the sensitivity estimation (light blue curve).

References

[1] Li, T.-P. & Ma, Y.Q. 1983, ApJ, 272, 317

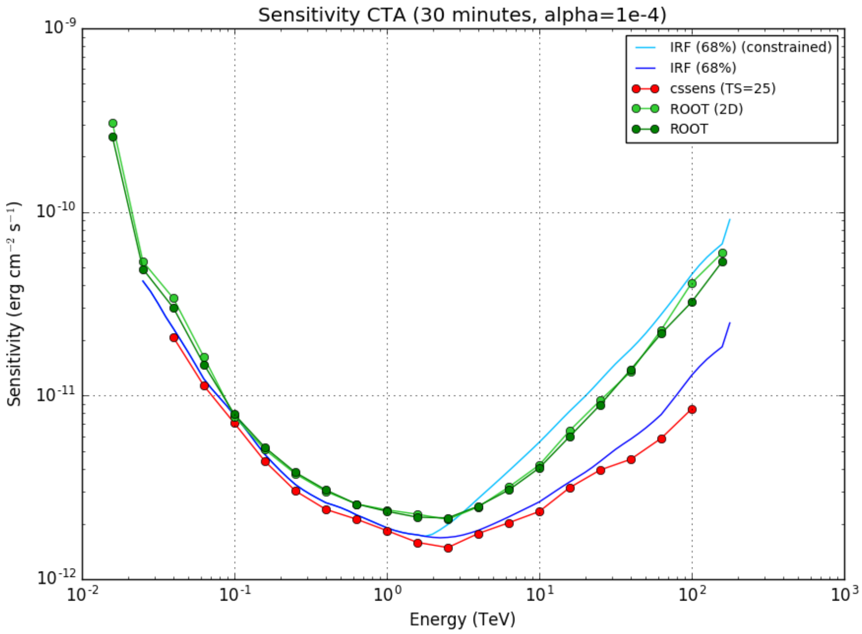

Figure 4: Comparison of sensitivities for 30 minutes cuts with $\alpha = 10^{-4}$ for the IRF sensitivities (blue curves).