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A MODEL OF THE BRIGHTNESS OF MOONLIGHT

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ABSTRACT

A knowledge of the brightness of moonlight is needed for detailed calculations of the limiting magnitude of astronomical detectors, whether they be visual, photographic, or electronic. The previous literature contains no method for making even crude estimates and has few actual measurements of moonlight brightness. In this paper we report new measurements of the sky brightness from the 2800-m level of Mauna Kea. In addition, we present a model for predicting the moonlight as a function of the Moon's phase, the zenith distance of the Moon, the zenith distance of the sky position, the angular separation of the Moon and sky position, and the local extinction coefficient. The model equations can be quickly calculated on a pocket calculator. A comparison of our model with our lunar data and with some Russian solar data shows the accuracy of our predictions to range from 8% to 23%.

Key words: night-sky brightness–moonlight–atmospheric extinction

1. Introduction

It is of fundamental interest to know the limiting magnitude of visibility for finding objects with a telescope. This subject has recently been addressed by Schaefer 1990a based on a database of 314 visual observations of stars viewed in small- to-medium-sized telescopes with a variety of magnifications. For large telescopes using video acquisition and guiding systems, the question is still important. It is also relevant to the prediction of proper exposure times for optical CCD cameras. A related question is: what is the magnitude limit when the Moon is out? With the eye, video system, or CCD camera the detection of an object depends on the contrast of the brightness of the object in question versus the brightness of the sky. We can think of this as achieving a certain signal-to-noise ratio. Whatever brightens the sky degrades the limiting magnitude of the eye or video system by approximately the same amount.

In a recent paper (Krisciunas 1990) we presented a sample of photometric measurements of the sky brightness, with and without moonlight. Most of the measurements (and *all* of the moonlight observations) were obtained at the 2800-m level of Mauna Kea. The telescope is a 15-cm Newtonian reflector, and the photometer employs standard *UBV* filters and an uncooled RCA 931A photomultiplier. The beam size is 6.5 square arc minutes.

At a site not hampered by light pollution it is found that the inherent dark sky level at the zenith (but away from the galactic plane and more than two hours after the end of twilight) is about $V = 21.8$ to 22.0 mag sec^{-2}, $B = 22.8$ to 23.0 mag sec^{-2}. At solar maximum these inherent levels are 0.4 to 0.6 mag sec^{-2} brighter.

The lunar sky brightness is the arithmetic difference between the observed sky brightness with the Moon above the horizon and the inherent dark sky value, given the phase of the solar cycle. In Figure 8 of Krisciunas 1990 these differences are plotted for 21 measurements, where the nominal dark sky levels are taken to be the yearly averages for the zenith. A brief summary of some measurements is also given by Walker 1987 based on measurements at Cerro Tololo. Walker's measurements made within 4 days of full Moon were taken 90 degrees or more away from the Moon.

The effect of the Moon on the sky brightness at some position is a function of the phase of the Moon, the zenith distance of the Moon, the zenith distance of the sky position, the angle between the Moon and the sky position, and the atmospheric extinction. In this paper we report additional moonlight brightness observations from the 2800-m level of Mauna Kea. In addition, we derive a simple model that gives the relation between the scattered moonlight and the five variables mentioned above. This model results in an easy-to-use equation that will predict the moonlight brightness with an accuracy of 8% to 23%.

2. Observations

A dozen observations of the brightness of the moonlit sky were made from the 2800-m level of Mauna Kea with the identical instrumental setup as used by Krisciunas 1990. These dozen observations are to be joined with the 21 observations in Table 5 of Krisciunas 1990. In Table 1 we give the new measurements of the observed V-band sky brightness on moonlit nights. The first six columns give the UT date and UT time of the observation, the azimuth and zenith distance of the sky position, and the azimuth and zenith distance of the Moon. Note that all angles have been rounded to the nearest degree since greater accuracy is not needed. The seventh column gives ρ, which is the angular distance between the Moon and the sky position. The eighth column gives the phase angle (α) of the Moon, which is defined as the angular distance between the Earth and the Sun as seen from the Moon. (When the Moon is between full and new, $\alpha < 0$.) The phase angles were calculated using the simpler algorithm of Meeus 1980 (eq. (31.4)), which ignores the nonconstancy of the (ecliptic) latitude of the Moon. The ninth column gives the observed V magnitude of the sky as measured in magnitudes per square arc second.

The last column in Table 1 gives the sky brightness of the moonlight alone as measured in nanoLamberts (nL). The reason for converting from a logarithmic unit (magnitudes per square arc second) to a linear unit (nanoLamberts) is because a magnitude difference compares the moonlight to the background which is variable from site to site. In addition, the use of a linear unit allows for the easy subtraction of the background sky brightness. The choice of the nanoLambert as the linear unit is to follow the lead of Knoll, Tousey & Hulburt 1946, Weaver 1947, Hecht 1947, Garstang 1989, and Schaefer 1990a. The relation between the sky brightness in nanoLamberts (B) and in magnitudes per square arc second (V) is

$$B = 34.08 \exp(20.7233 - 0.92104\, V) \ , \qquad (1)$$

as given in equation (27) of Garstang 1989. The surface-brightness equivalent to one tenth magnitude star per square degree (that is, the S10 unit) equals 0.263 nL. An alternative expression, equivalent to equation (1), which includes the definition of the S10 unit and the scale factor to convert to nanoLamberts, is

$$B = 0.263\, a^{(Q-V)} \ ,$$

where $a = (100)^{0.2} \approx 2.51189$ and $Q = 10.0 + 2.5 \log(3600^2) \approx 27.78151$.

The dark nighttime sky brightness (B_0) as a function of zenith distance (Z) is

$$B_0(Z) = B_{\rm zen}\, 10^{-0.4\,k(X-1)}\, X \ , \qquad (2)$$

where

$$X = (1 - 0.96 \sin^2 Z)^{-0.5} \ . \qquad (3)$$

The extinction coefficient is k in units of magnitudes per air mass. X is the optical pathlength along a line of sight in units of air masses. Note that the formula for X in equation (3) is appropriate for the night glow. Equations (2) and (3) are simplifications of equations (29) and (30) from Garstang 1989 that are adequate for the present purposes. The dark zenith sky brightness ($B_{\rm zen}$) is taken as the yearly average reported in Krisciunas 1990 or as discussed below. The extinction coefficients were taken to equal the value measured for that night or the median value at the 2800-m level of Mauna Kea, namely 0.172 mag/air mass, obtained by Krisciunas 1990.

The last column of Table 1 has the background sky brightnesses subtracted out. That is, the value for the background alone, $B_0(Z)$, is subtracted from the observed brightness for the moonlight plus background, B, leaving the brightness of the moonlight alone, $B_{\rm moon}$. When the moonlight is dim compared to the background, the natural variations in the background will cause large relative errors in derived values of $B_{\rm moon}$. Therefore, we have excluded nine observations (for which the moonlight is fainter than the dark sky background) from our statistical analysis of the model compared to actual observations.

The observations of 1991 January 4 (UT) are particularly useful, since a concerted effort was undertaken to narrow down the functionality of the components of the lunar sky-brightness effect. This involved obtaining a dark-sky value on that night ($V = 21.444$ mag sec^{-2}) an hour before the Moon rose. This value compares well with the 1990 average (the year of sunspot maximum) of $V = 21.377 \pm 0.095$ mag sec^{-2}, based on measurements of 5 nights. For the two observations in Table 1 from 1990 we shall adopt the 1990 yearly average as the nominal dark-sky value. We shall adopt the *actual* dark-sky value

TABLE 1

V-band sky brightness on nights with moonlight[a]

UT Date	UT Time	Sky posn. AZ °	Sky posn. Z °	Moon AZ °	Moon Z_m °	ρ °	α °	V mag/sec^2	B_{moon} (Obs.) nL
1990									
12 Feb	7:11	280	9	94	79	88	-30	19.813	308
28 Dec	7:05	89	18	300	4	22	49	18.907	833
1991									
4 Jan	8:07	125	17	85	82	68	-48	20.141	206
4 Jan	8:15	63	38	85	80	45	-48	19.605	380
4 Jan	8:23	55	30	86	78	53	-48	19.684	354
4 Jan	8:30	82	52	87	77	24	-49	18.615	1087
4 Jan	8:34	143	13	87	76	69	-49	19.902	281
4 Jan	8:42	99	61	87	74	16	-49	18.456	1255
4 Jan	8:49	79	68	88	72	9	-49	18.228	1557
4 Jan	8:54	165	11	88	71	69	-49	19.771	329
21 Jan	6:26	263	49	265	62	14	119	20.572	71
21 Jan	6:33	113	24	266	64	85	119	21.031	30

[a]Observations made at 2800-m elevation of Mauna Kea, Hawaii.

for 1991 January 4 for reference in calculating the lunar effect on that night. On this night we also measured the V-band extinction to be $k = 0.116 \pm 0.019$ mag/air mass, based on 8 measurements of 5 stars over a range of 1.5 air masses. For the 1991 January 21 data we shall adopt a dark-sky value of $V = 21.388$ mag sec^{-2}, equal to the average of the six measurements of the previous year.

3. Atmospheric Scattering

The atmospheric scattering of moonlight is a complex problem. The sky brightness will depend on the extinction coefficient, the magnitude of the Moon, the zenith distance of the Moon, the angular separation between the Moon and the sky position, and the zenith distance of the sky position. In these next two sections we describe a simple scattering model (elaborated by one of us, B.E.S.) so as to establish functional forms for the variation of each parameter that are fairly accurate.

When looking out along a line of sight, an observer will detect scattered light from the Moon. Let the field of view be a circle with an angular radius of ϕ, so that the scattered light must come from inside a narrow cone. The volume of the cone can be broken up into differential volumes

$$dV = \pi\,(R\,\phi)^2\,dR \quad , \tag{4}$$

where R is the distance from the observer to the volume. This volume will have some density of scatterers, D, which will be proportional to the volume extinction coefficient b

$$D = cb \quad , \tag{5}$$

where c is some constant. The number of scatterers in the differential volume, dN, will be

$$dN = D\,dV = \pi\,c\,b\,R^2\phi^2 dR \quad . \tag{6}$$

The brightness of scattered light from each volume will be proportional to dN.

The differential volume will be illuminated by moonlight dimmed by the atmosphere. If I^* is the illuminance of the Moon outside the atmosphere and I is the illuminance of the Moon inside the atmosphere, then these two quantities will be related as

$$I = I^*\,10^{-0.4\,k\,X_{\rm m}} \quad , \tag{7}$$

where k is the extinction coefficient and $X_{\rm m}$ is the optical pathlength to the Moon as measured in air masses (see below). The illuminance of the Moon (in footcandles) can be related to the V magnitude of the Moon (m) as

$$I^* = 10^{-0.4\,({\rm m}\,+\,16.57)} \tag{8}$$

(see eq. (16) of Schaefer 1990a). The magnitude of the Moon will be a function of the phase angle, α in degrees, such that

$$\mathrm{m} = -12.73 + 0.026\,|\alpha| + 4 \times 10^{-9}\,\alpha^4 \tag{9}$$

(see page 144 of Allen 1976). The brightness of scattered moonlight will be proportional to I. Equation (9) gives m $= -12.73$ for full Moon, in agreement with the results of Lane & Irvine 1973 *extrapolated* to $\alpha = 0$. However, this ignores the "opposition effect". For $|\alpha| < 7°$ the brightness of the Moon deviates from this relation (see, e.g., Whitaker 1969). When the Moon is exactly full it is about 35% brighter than the extrapolation would predict, assuming, of course, that it is not undergoing a penumbral or umbral eclipse.

The luminous intensity of the differential volume, dL, will be proportional to the number of scatterers in the volume, the illuminance of the Moon, and the scattering function. The scattering function, $f\,(\rho)$, describes the intensity of scattered light as a function of the scattering angle ρ. Therefore,

$$dL = I\,f(\rho)\,dN \quad . \tag{10}$$

In this equation and below, the various constants of proportionality will be absorbed into the scattering function. The light traveling from the differential volume to the observer will be attenuated not only by the atmosphere but also by the inverse square law for the intensity of light. The perceived surface brightness from the differential volume, dB, will be the perceived brightness divided by the subtended solid angle, so that

$$dB = dL\,e^{-\tau}/\pi\phi^2R^2 = I\,f(\rho)\,b\,e^{-\tau}\,dR \quad . \tag{11}$$

Here, τ corresponds to the optical depth between the observer and the volume.

The total apparent surface brightness can be found by integrating along the entire line of sight. For the simple model that the atmosphere is uniform up to some height H, then the optical depth will be simply bR. The integral must run from zero to the distance of the top of the atmosphere (that is $H\sec(\mathrm{Z})$). Thus, the surface brightness will be

$$B = I\,f(\rho)\,[1 - e^{-bH\sec(\mathrm{Z})}] \quad . \tag{12}$$

For such an atmospheric model, the extinction coefficient can be related to the volume extinction coefficient as

$$10^{-0.4\,k} = e^{-bH} \quad . \tag{13}$$

The air mass is often evaluated as the secant of the zenith distance, and this is accurate for any direction not near the horizon. So, the secant of the zenith distance in equation (12) will be better approximated as the air mass along the line of sight, $X(\mathrm{Z})$.

Many expressions for air mass that are valid low on the horizon have been proposed (Hardie 1962; Rozenberg 1966; Garstang 1989; Schaefer 1989). Ideally, each extinction component must be handled separately since they all have different height distributions (cf. Schaefer 1990b).

From these correct calculations, we have found that equation III.3, 17 from Rozenberg 1966 is the most reasonable of the simple formulae for zenith distances all the way to the horizon. Therefore,

$$X(Z) = [\cos(Z) + 0.025\, e^{-11 \cos(Z)}]^{-1} \ . \qquad (14)$$

In this formula the air mass goes as sec(Z) when far from the horizon but is limited to 40 air masses on the horizon. This is the best formula to use for correcting the brightness of an object *directly* observed (e.g., the Moon itself). However, for *scattered light* from the Moon we find that air masses based on equation (3) give the better fit of observed lunar brightness vs. model brightness, for a very large range of $B_{\rm moon}$. We might call equation (3) the "scattering air mass" and equation (14) the "extinction air mass", and it is clear from our data that equation (14) leads to a gross underestimate of the lunar sky-brightness effect when the Moon is low on the horizon.

From equations (7), (12), and (13), the model surface brightness will be

$$B_{\rm moon} = f(\rho)\, I^*\, 10^{-0.4\, kX(Z_{\rm m})}\, [1-10^{-0.4\, kX(Z)}] \ . \qquad (15)$$

The evaluation of $f(\rho)$ will be given in the next section.

This last equation embodies a number of complexities, the details of which have not been fully elaborated. The most obvious are:

(1) The volume extinction coefficient b will be a sum of two exponential functions of height.

(2) The extinction in equation (7) will be a function of the height of the volume.

(3) The effects of a spherical Earth and refraction become important near the horizon. Better calculations can and have been made (cf. Rozenberg 1966 and references therein). However, the results are of much greater complexity.

(4) The scattered light has a small but significant fraction of multiply scattered light, whereas the use of the scattering function in equation (12) implicitly assumes single scattering.

Equation (15) has also ignored several effects, some of them quite minor:

(1) The Moon has an asymmetric distribution of maria so that the Moon and, hence, the scattered moonlight should be slightly brighter before full Moon than after (Stebbins & Brown 1907; Russell 1916; Rougier 1934). However, this effect is very small and Lane & Irvine 1973 were unable to detect it.

(2) For $|\alpha| < 7°$ the lunar opposition effect should be taken into account, in which case the derived value of $B_{\rm moon}$ must be multiplied by a factor in between 1.00 and 1.35. This factor only comes into play within a day of full Moon.

(3) Refraction will change the altitudes and separation of the Moon and the sky position from those used in this model. However, even on the horizon, the altitude will only shift by half a degree at most and will change the predicted brightness by only small amounts.

(4) The position of the Moon as used in this paper is that of the Moon's center, whereas it would be more accurate to use the position of the light centroid. However, under extreme conditions, the difference in position is a quarter of a degree at most, so that this effect is negligible.

(5) The Moon is sometimes closer and, hence, larger and brighter than at other times for the same phase angle. The apparent diameter of the Moon will vary by 11.6% on average from apogee to perigee, so that the Moon's magnitude, as derived by equation (9), may be in error by as much as $\pm$ 0.12 magnitude.

An improved model would account for all of the above effects and more. Yet the real atmosphere is never as well behaved as in any model ever calculated. In particular, the concentration and distribution of the atmospheric aerosols are highly variable with time and location (cf. Fig. 56 of Rozenberg 1966). With such large variations, it is useless to refine a model intended for general application.

The real justification for the simplicity of the model in equation (15) is that it reproduces the observations with remarkable accuracy. We will show below that the model reproduces the Mauna Kea data (from scattered moonlight) with an rms uncertainty of 23% and the data (from scattered sunlight) from two Russian sites (Pyaskovskaya-Fesenkova 1957) to an accuracy of 8% to 11%. This accuracy is achieved despite the hundredfold variation in the observed brightness of scattered moonlight.

4. Scattering Function

Equation (15) gives a simple formula for the moonlight brightness, where the functional forms for I^*, k, $X_{\rm m}$, and X are derived from theory. However, the scattering function, $f(\rho)$, has not yet been evaluated. The scattering function, $f(\rho)$, is proportional to the fraction of incident light scattered into a unit solid angle with a scattering angle ρ. The scattering angle, ρ, will be equal to the angular separation between the Moon and the sky position for single scattering.

The scattering function will be composed of additive terms associated with the two types of scattering in our atmosphere. The first type is Rayleigh scattering from atmospheric gases, which will contribute $f_{\rm R}(\rho)$. The second type is Mie scattering by atmospheric aerosols, which will contribute $f_{\rm M}(\rho)$. The two terms will add (see eq. II.2, 38 of Rozenberg 1966), so that

$$f(\rho) = f_{\rm R}(\rho) + f_{\rm M}(\rho) \ . \qquad (16)$$

Remember that the scattering functions have absorbed constant factors relating to unit conversions and normalizations.

For Rayleigh scattering from atmospheric gases, the contribution to the scattering function will be

$$f_R(\rho) = C_R\,[1.06 + \cos^2(\rho)] \qquad (17)$$

(Rozenberg 1966). C_R is a proportionality constant yet to be determined.

For Mie scattering of aerosols, the contribution to the scattering function will be $f_M(\rho)$. Explicit equations for $f_M(\rho)$ have been compiled in the treatise of van de Hulst 1957 for many particle shapes, sizes, and optical properties. Unfortunately, these idealized cases have no utility for the real atmosphere where the aerosol size distribution is broad, and the particle shape and composition are widely variable. In principle, $f_M(\rho)$ can be found by integrating the single particle functions over the size, shape, and optical properties of the aerosols, but in practice this knowledge is never known with sufficient accuracy to justify the integration. Henyey & Greenstein 1941 offer a scattering function that is merely a convenient functional form with no basis in data. Thus, the only alternative is that of empirical measurement.

Therefore, in practice, we will use the observations from the previous section to define the scattering function. This can be done because B_{moon}, I^*, k, X_m, and X are known for all observations from Mauna Kea, so that equation (15) can be solved for $f(\rho)$. Figure 1 shows a plot of the scattering function as a function of the Moon/sky separation.

Pyaskovskaya-Fesenkova 1957 presents smoothed data for the daytime sky brightness that can also be used to evaluate $f(\rho)$. The reason is that scattered sunlight is identical in everything but intensity with scattered moonlight. The difference in intensity can be easily accounted for by using the Sun's magnitude ($m_{sun} = -26.74$) instead of the Moon's magnitude in equation (8). The Russian data are for the whole sky from sites with $k = 0.15$ and $k = 0.24$ and for solar zenith distances of 0, 30, 60, and 80 degrees. We have extracted many representative points, calculated $f(\rho)$ from equation (15), and plotted the results in Figure 2.

The first item to notice is that the scattering function is identical (to within the scatter) for the three sites with extinction coefficients of 0.15, 0.172, and 0.24. Second, there is no systematic difference between solar and lunar data. The third and fourth items are that the data with the zenith distance of the source and of the sky position greater than 80 degrees are not systematically different from the average. These four items show that the functional dependence on k, I^*, X_m, and X is correct. This is the empirical justification that the approximations leading to equation (15) are good.

The Mie scattering by aerosols is highly forward scattering. Thus, the Rayleigh scattering will dominate for scattering angles greater than 90 degrees. Therefore, the empirical measures of $f(\rho)$ for large angles can be used to measure C_R. The value will be the average (over data values with a scattering angle of greater than 90 degrees) of the empirical $f(\rho)$ divided by $1.06 + \cos^2(\rho)$. We find a value of 2.27×10^5 for C_R.

The Rayleigh component of $f(\rho)$ can then be subtracted out, leaving only $f_M(\rho)$. For scattering angles from 10 to 80 degrees, the empirical $f_M(\rho)$ is roughly a straight line on a

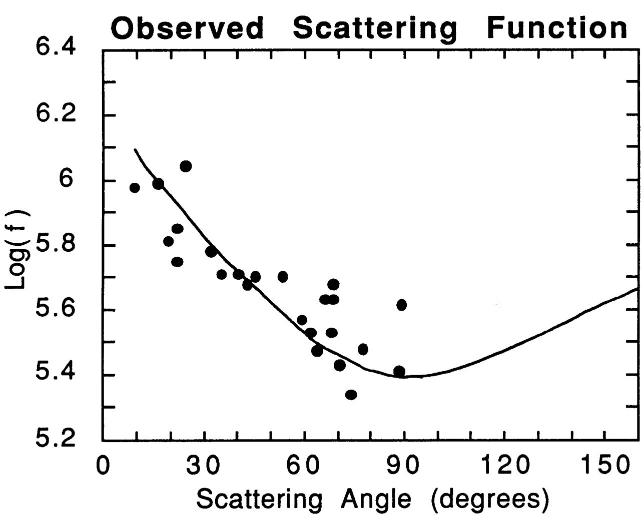

FIG. 1–The scattering function, $f(\rho)$, as deduced from the moonlight observations reported in this paper and in Krisciunas 1990. That is, for the observations, all the quantities in equation (15) are known so that the scattering may be solved for. The scattering function is a function of the scattering angle, ρ. The scattering function from equation (21) is drawn as the smooth curve.

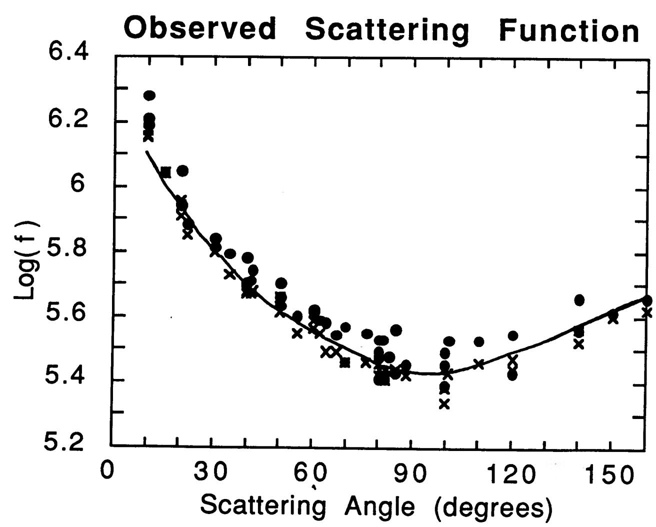

FIG. 2–The scattering function, $f(\rho)$, as deduced from the sunlight observations reported in Pyaskovskaya-Fesenkova 1957. As in Figure 1, all the quantities in equation (15) are known so that the scattering function may be determined. The scattering function is plotted versus the scattering angle, ρ, for sites with extinction coefficients of 0.15 (crosses) and 0.24 (dots). The scattering function from equation (21) is drawn as the smooth curve. Note that the scatter about the model is small and that there is no significant difference between the two sites, as this is the empirical justification that the functional form of equation (15) is valid.

log-linear plot. From this plot we will adopt a formula for

$$f_M(\rho) = 10^{6.15-\rho/40} \ . \tag{18}$$

Now the total scattering function can be found as a function of ρ from equations (16), (17), and (18), and is displayed as a curve in Figures 1 and 2.

For angular separations less than 10 degrees, the data in King 1971 show that equation (18) is not valid. Schaefer 1991 has shown that

$$f_M(\rho) = 6.2 \times 10^7 \rho^{-2} \ . \tag{19}$$

The scattering angle is measured in degrees for both equations (18) and (19). This small angle scattering forms the aureole.

The reader may wish to compare our scattering function, using appropriate scaling, to that of McClatchey et al. 1978 (Fig. 26 in Section 14) and that of Rozenberg 1966 (Fig. 72).

5. Discussion

The final equation for $B_{\rm moon}$ is derived from equations (3), (8), (9), and (15)–(19). For Moon/sky separations greater than 10 degrees, these results can be summarized as

$$I^* = 10^{-0.4(3.84 + 0.026|\alpha| + 4\times 10^{-9}\alpha^4)} \ , \tag{20}$$

$$f(\rho) = 10^{5.36}[1.06 + \cos^2(\rho)] + 10^{6.15-\rho/40} \ , \tag{21}$$

$$X(Z) = (1 - 0.96\sin^2 Z)^{-0.5} \ , \tag{3}$$

$$B_{\rm moon} = f(\rho)\, I^*\, 10^{-0.4\,kX(Z_m)}[1 - 10^{-0.4\,kX(Z)}] \ . \tag{15}$$

The required input for this general formulation is the lunar phase angle α, the Moon/sky separation ρ, the extinction coefficient k, the zenith distance of the Moon Z_m, and the zenith distance of the sky position Z. These equations are not as formidable as they might look. A pocket calculator can evaluate $B_{\rm moon}$ in about a minute and only a few lines of code are needed in a computer program.

The change in the V-band sky brightness caused by the moonlight will be

$$\Delta V = -2.5\log[(B_{\rm moon} + B_0(Z))/B_0(Z)] \ . \tag{22}$$

As a guide, we have tabulated typical values of $B_{\rm moon}$ (in nL) and ΔV (in magnitudes per square arc second) for a range of α and ρ (see Table 2). This table was constructed with the assumption that k = 0.172 mag/air mass (the median V-band extinction at the 2800-m level of Mauna Kea), and that $V_{\rm zen} = 21.587$ mag sec^{-2} (the mean dark-sky value at the same site, equivalent to $B_{\rm zen}$ = 79.0 nL). Strictly speaking, the values of $B_{\rm moon}$ and ΔV in Table 2 only apply to a site with the same median extinction coefficient and mean dark-sky brightness as the 2800-m level of Mauna Kea (e.g., McDonald Observatory).

TABLE 2

Lunar sky brightness effect from model[a]

Phase angle (α)	Angular distance between moon and sky position (ρ) 5°	30°	60°	90°	120°
30°	7216 (-4.48)	1160 (-2.87)	530 (-2.22)	437 (-1.93)	818 (-2.16)
60°	3364 (-3.67)	541 (-2.13)	247 (-1.54)	204 (-1.30)	381 (-1.49)
90°	1351 (-2.73)	217 (-1.35)	99 (-0.88)	82 (-0.71)	153 (-0.85)
120°	391 (-1.58)	63 (-0.58)	29 (-0.34)	24 (-0.26)	44 (-0.32)

[a]Values of $B_{\rm moon}$, measured in nanoLamberts. For lunar zenith angle of 60° and V-band extinction of 0.172 mag/airmass. For this table ρ is measured along the great circle on the sky passing through the moon and the zenith. Therefore, the column ρ = 60° corresponds to the zenith. Values in parentheses are ΔV in mag/sec^2, using V = 21.587 mag/sec^2 ($B_{\rm zen}$ = 79.0 nL) as the nominal zenith sky brightness, and scaling the zenith sky brightness to the nominal value at the zenith angle corresponding to ρ.

Given the two extinction terms in equation (15), a table comparable to Table 2 for the Mauna Kea summit (with median V-band extinction of 0.113 mag/air mass; Krisciunas 1990) gives $B_{\rm moon}$ values which are all lower. The extreme example would be for a satellite in Earth orbit, for which $k = 0$ and $B_{\rm moon}$ always equals 0 (i.e., the sky is black). For a dark sky site with $k = 0.30$ the sky-brightness effect due to the Moon is larger than those values tabulated in Table 2. Because the moonlight contribution comes from scattering of light, where one is observing through more atmosphere (i.e., sea level vs. on a mountaintop), there will be more scattering.

For Palomar and Mount Wilson, which have V-band extinction comparable to that at the 2800-m level of Mauna Kea, but different dark-sky brightnesses (due to artificial light), one can use the values of $B_{\rm moon}$ from Table 2 and equations (1), (2), (3), and (22), with an appropriate value of $V_{\rm zen}$ for equation (1), to calculate the sky-brightness effect (ΔV) in mag sec^{-2}.

One of the principal uncertainties in using equation (22) on any given night comes from the uncertainty of the adopted dark-sky zenith brightness. Even if it is measured, one does not know the exact contribution of faint stars in the beam or what the effect is of the solar wind on the air glow on that specific day. See Krisciunas 1990 for further discussion.

The accuracy of our general formulation can be determined from the scatter of the observed brightnesses about the predictions. In Figure 3 we plot $B_{\rm moon}$ (ob-

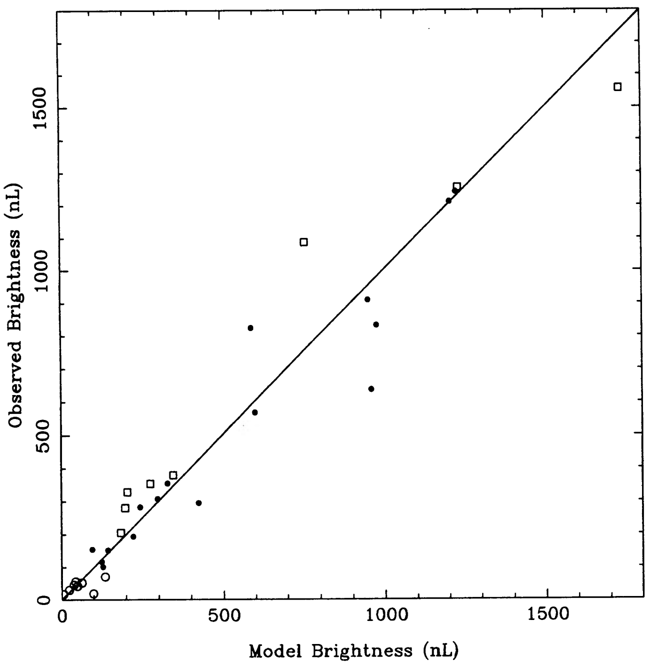

FIG. 3–Observed values of sky brightness due to scattered moonlight (in nanoLamberts) vs. model values from equation (15). Open circles: data for which $B_{moon} < B_0(Z)$. Squares: data of 1991 January 4. Large dots: all other data. A line of slope 1 is indicated.

served) vs. B_{moon} (model) for the data of Krisciunas 1990 and from Table 1 of this paper. If we take the ratios

$$[B_{moon}\,(\text{observed}) - B_{moon}\,(\text{model})]/B_{moon}\,(\text{observed})$$

we find an rms variation of this ratio of 23% for the Mauna Kea data (with $B_{moon} > B_0(Z)$), 8% for the $k = 0.15$ Russian site, and 11% for the $k = 0.24$ Russian site. Hence, we conclude that our formula has an accuracy of 8% to 23% in the prediction of the sky brightness from moonlight. The accuracy will be poorer when the Moon or sky position is close to the horizon so that small uncertainties in the extinction coefficient will be magnified.

To the best of our knowledge, no formula for moonlight brightness has ever appeared in the literature. Our formula has the advantages of having the correct functional dependencies, yet being easy to use and accurate to better than 23%. As such, the equations can be used to optimize exposure times for photography and CCD images, to predict limiting magnitudes for visual observations, as well as to predict the visibility of stars, planets, or comets near the Sun or Moon for historical purposes.

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