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An analytical formula for the time transformation TB–TT

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Abstract. An analytical formula for the time transformation TB–TT valid over a few thousand years around J 2000 has been computed with an accuracy at the 1 ns level. The 127 coefficients presented in this paper provide a formula accurate at the 100 ns level. The numerical and analytical procedures to compute this transformation are discussed and compared. We note that these procedures cannot fully comply with recommendation 5 of the 1976 IAU meeting. Furthermore, these procedures yield different units for the corresponding TB time scales. We have verified that this transformation is independent of the two Parametrized Post Newtonian (PPN) parameters γ and β and of the three most commonly used coordinate systems (isotropic, standard-Schwarzschild, Painlevé) at least at the 1 ns level.

Key words: astrometry – celestial mechanics – time scales

1. Introduction

Recommendation 5 of the 1976 IAU meeting in Grenoble (see Winkler and Van Flandern, 1977) states that:

the time scales for equations of motions referred to the barycentre of the solar system be such that there be only periodic variations between these time scales and that for the apparent geocentric ephemerides.

The exact IAU denominations for these two time scales are "Terrestrial Dynamical Time" (TDT) and "Barycentric Dynamical Time" (TDB) but we shall adopt the abbreviations "TT" and "TB", respectively, as suggested by Guinot and Seidelmann (1988) and later proposed to the Working Group on Reference Frames at the IAU XX General Assembly in Baltimore (1988).

The complete formulation for transforming the physically realized time scale of a clock on the surface of the Earth to the corresponding time in Barycentric Time can be found in Guinot (1986). It includes corrective terms comparing the clock to TAI, a diurnal term depending on the location of the clock on the Earth (giving the transformation TAI–TT) and a periodic term depending on the position of the Earth relative to all the solar system bodies. It is this latter periodic term that gives the transformation from TT to TB and that will be developed analytically in this paper.

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A new impetus for the need of an accurate time transformation between the time of an event read from an observatory clock and the corresponding coordinate time in Temps Barycentrique (TB) is the analysis of timing data of millisecond pulsars, e.g. PSR 1937+214. At present, the precision of these data acquired at Arecibo and at Nançay is below 1 µs and is expected to improve to 0.1 µs. Hence, the physical model required to analyse these observations must include a time transformation which is precise at the 0.01 µs level (one tenth of the expected observation error). We have finally set the requirement that our TB–TT transformation be accurate to the 1 ns level for future applications. The transformation TAI–TT can be computed at this level too. Comparison of time between observatory clocks by space techniques (GPS, LASSO, use of communication satellites) are approaching this level of accuracy.

There are two possible procedures to calculate the time transformation TB–TT. The first procedure is numerical: the differential equation giving TB as a function of TT is integrated numerically over an interval of approximately one century to produce a "time ephemeris" that will drift linearly. This linear trend is computed by averaging the "time ephemeris" over the integration interval and is then subtracted from the "time ephemeris" itself. The resulting tabulated values provide a time transformation TB–TT which matches only to some extent the IAU recommendation that only periodic variations be kept. The second procedure is based on analytical theories for the motions of the planets and Moon. The planetary theory VSOP 82 (Bretagnon, 1982) and the lunar theory ELP 2000 (Chapront-Touzé and Chapront, 1983) developed at the Bureau des Longitudes can be used to calculate an analytical formula for the TB–TT transformation as presented below.

2. An analytical formula for the time transformation TB–TT

The Parametrized Post Newtonian (PPN) metric given by Eq. (4) in Brumberg (1986) describes the space-time properties of the solar system. This metric includes the contributions of all the planets and their mutual interactions. It also includes the two physical PPN parameters γ and β and the integer ν (with possible values 0, 1, 2) for selecting one of the three most commonly used coordinate systems (isotropic, standard Schwarzschild, Painlevé) in celestial mechanics.

This complete metric was used to derive the differential relation between TT and TB. The resulting expression is rather voluminous. However it turns out that the only terms which need be kept to have a formula providing 1 ns accuracy after integration lead to the differential expression:

$$\frac{\mathrm{d\,TT}}{\mathrm{d\,TB}} = 1 - \left(\sum_i \frac{m_i}{\varrho_i} + \frac{1}{2} \left(\frac{v}{c} \right)^2 \right), \quad (1)$$

where $m_i = GM_i/c^2$, M_i mass of planet i, $\varrho_i = \boldsymbol{r} - \boldsymbol{r}_i$, $\boldsymbol{r}_i$ barycentric position of planet i and $\boldsymbol{r}$ and v stands for the barycentric position and velocity of the Earth, respectively.

It is interesting to note that differential Eq. (1) does not depend explicitly either on the two physical PPN parameters γ and β or on the integer v selecting the coordinates. Hence, at the 1 ns level, the time transformation TB–TT is independent of the three systems of coordinates considered by Brumberg (1986).

Equation (1) is given by Thomas (1975) and by Moyer (1981) who provides an analytical solution accurate to 20 µs. Hirayama et al. (1988) provide an analytical solution which is more precise and to which we compared our results.

Equation (1) is integrated to provide the TB–TT transformation. The integration can be a numerical or an analytical one. We have integrated (1) analytically and have used the Bureau des Longitudes ephemerides VSOP 82 and ELP 2000. The analytical theories VSOP 82 and ELP 2000 for the motions of the planets and Moon have integration constants which are adjusted on the Jet Propulsion Laboratory ephemeris DE 200 (Newhall et al., 1983) but with the IAU recommended values for the planetary masses. These analytical theories provide ϱ_i and v as trigonometric time series. Using these in (1) and keeping all periodic terms larger than 10^{-11} before integration to insure 1 ns accuracy, the resulting formula after integration is in the form:

$$\begin{aligned} \mathrm{TT} = {} & (1+\tilde{B}_0)\mathrm{TB} + \tilde{C}_0 \mathrm{TB}^2 + \dots \\ & + \sum_i \tilde{A}_i \sin(\tilde{\omega}_{ai}\mathrm{TB} + \phi_{ai}) + \mathrm{TB} \sum_i \tilde{B}_i \sin(\tilde{\omega}_{bi}\mathrm{TB} + \phi_{bi}) \\ & + \mathrm{TB}^2 \sum_i \tilde{C}_i \sin(\tilde{\omega}_{ci}\mathrm{TB} + \phi_{ci}) + \mathrm{TB}^3 \sum_i \tilde{D}_i \sin(\tilde{\omega}_{di}\mathrm{TB} + \phi_{di}) . \end{aligned} \quad (2)$$

We wish, in fact, to express TB as a function of TT. Equation (2) must therefore be inverted. This has been done. To conform to the IAU convention that no linear term be present in the transformation and to insure that the two time scales have the same unit of time, we then divided both sides of the resulting equation by $(1-\tilde{B}_0)$ where $\tilde{B}_0 = -467.313\ \mathrm{s}/10^3\ \mathrm{yr}$. The transformation has the final form:

$$\begin{aligned} \Delta\mathrm{T} = {} & C_0 \mathrm{TT}^2 + \dots \\ & + \sum_i A_i \sin(\omega_{ai}\mathrm{TT} + \phi_{ai}) + \mathrm{TT} \sum_i B_i \sin(\omega_{bi}\mathrm{TT} + \phi_{bi}) \\ & + \mathrm{TT}^2 \sum_i C_i \sin(\omega_{ci}\mathrm{TT} + \phi_{ci}) + \mathrm{TT}^3 \sum_i D_i \sin(\omega_{di}\mathrm{TT} + \phi_{di}) \end{aligned} \quad (3)$$

where TT on the right hand side is in thousands of years from J 2000.0, ΔT, the difference TB–TT, is in microseconds and the coefficients are given in Table 1. The column marked Period in Table 1 gives the period of the term in years. The arguments of each term of Table 1 are a combination of the mean longitudes of the planets of the solar system and the Moon. The columns headed arguments indicate the contribution of each planet to the term. The series has been truncated at the 10 ns level to provide the 127 coefficients given in Table 1. This insures a TB–TT transformation with an accuracy of 100 ns. We have also computed an analytical transformation precise at the 1 ns level by calculating all terms greater than 0.1 ns. This level of accuracy is attained with 750 coefficients. A computational error caused some coefficients given in a previous paper (Fairhead, Bretagnon and Lestrade, 1987) to be in error. The main coefficient, for example, is erroneous by 15 ns.

3. Comparison with other formulae

The Jet Propulsion Laboratory (JPL) and the Centre for Astrophysics (CfA) have established numerical TB–TT transformations by integrating Eq. (1) numerically to obtain a "time ephemeris". They comply to the IAU recommendation by averaging their numerical "time ephemeris" over an interval of 100 yr for the JPL and 60 yr for the CfA and subtracting this average to the "time ephemeris". Over such small time intervals, the long period terms present in the ephemeris will be identified as linear terms in time. These long period terms will therefore be ignored by the numerical time transformation. On the contrary, they will be kept in the analytical formula. This difference will amount to a change of unit between the TB time scales generated by analytical and numerical transformations.

We have compared our 1 ns analytical formula to these two numerical transformations. As expected, the comparisons show (Fig. 1), a linear difference between the analytical formula and the numerical ones. These drifts amount to 32 µs for 100 yr or $10^{-14}\ \mathrm{ss}^{-1}$ for the JPL's formula and to 0.5 µs for 60 yr or $3\ 10^{-16}\ \mathrm{ss}^{-1}$ for the CfA formula. These linear drifts cannot be neglected, particularly in timing of millisecond pulsars. The characteristics of these pulsars will allow the determination of their periods with a relative uncertainty better than 10^{-14}. Rawley (1986) has already determined the period of PSR 1937+214 with a relative uncertainty of $3\ 10^{-14}$. The use of different TB–TT transformations will therefore lead to differing values of the millisecond pulsars periods. Furthermore, and this has implications for other applications, the best atomic time standards already have relative frequency accuracies of $1.5\ 10^{-14}$ and these are improving. Thus, uncertainties in timing accurately any phenomena would arise from the TB–TT transformation used rather than from the atomic time standard with which the timing was accomplished.

After removing the linear slope from the differences between the JPL and our formula, we obtained the residuals shown in Figs. 2 and 3. The difference between these two figures is due to the values of the masses used for Saturn and Uranus. In Fig. 2, the ephemeris used to provide ϱ_i and v in Eq. (1) used the masses recommended by the IAU ($M_{\mathrm{Saturn}} = M_\odot/3498.5$, $M_{\mathrm{Uranus}} = M_\odot/22869$). In Fig. 3, the ephemeris used the masses adjusted by the JPL ($M_{\mathrm{Saturn}} = M_\odot/3498$, $M_{\mathrm{Uranus}} = M_\odot/22960$).

The comparison of the CfA's formula and the analytical formula shows short-term variations of the order of 200 ns whereas Figs. 2 and 3 show that short-term variations between the JPL transformation and the analytical formula are of the order of 3 ns, a level which is quite adequate for present accuracy needs. We cannot explain this discrepancy between the two comparisons.

Finally, we have compared the analytical formula given by Hirayama et al. (1987) and our own. The term with argument $8E-16M+4J+5S$ with period 93462 yr has been developed as a time polynomial in our formula as it is a long period term for the range of use (~1000 yr). The linear and quadratic terms were incorporated in the appropriate terms of Eq. (2). The long period terms with arguments $4E-8M+3J$ (period ≈1783 yr), $2J-6S+3U$ (period ≈1598 yr), $5V-6E-4M$ (period

Table 1. Table of the coefficients of Eq. (3) for the analytical transformation TB–TT

i	A_i (μs)	ω_{ai} (rd/10^3y)	ϕ_{ai} (rd)	Period (years)	Arguments Me	V	E	M	J	S	U	N	D	F	L
1	1656.674564	6283.075943033	6.240054195	1.0000	0	0	1	0	0	0	0	0	0	0	0
2	22.417471	5753.384970095	4.296977442	1.0921	0	0	1	0	-1	0	0	0	0	0	0
3	13.839792	12566.151886066	6.196904410	0.5000	0	0	2	0	0	0	0	0	0	0	0
4	4.770086	529.690965095	0.444401603	11.8620	0	0	0	0	1	0	0	0	0	0	0
5	4.676740	6069.776754553	4.021195093	1.0352	0	0	1	0	0	-1	0	0	0	0	0
6	2.256707	213.299095438	5.543113262	29.4572	0	0	0	0	0	1	0	0	0	0	0
7	1.694205	-3.523118349	5.025132748	1783.4159	0	0	4	-8	3	0	0	0	0	0	0
8	1.554905	77713.772618729	5.198467090	0.0809	0	0	0	0	0	0	0	0	1	0	0
9	1.276839	7860.419392439	5.988822341	0.7993	0	2	-2	0	0	0	0	0	0	0	0
10	1.193379	5223.693919802	3.649823730	1.2028	0	0	1	0	-2	0	0	0	0	0	0
11	1.115322	3930.209696220	1.422745069	1.5987	0	1	-1	0	0	0	0	0	0	0	0
12	0.794185	11506.769769794	2.322313077	0.5460	0	0	2	0	-2	0	0	0	0	0	0
13	0.600309	1577.343542448	2.678271909	3.9834	0	2	-3	0	0	0	0	0	0	0	0
14	0.496817	6208.294251424	5.696701824	1.0121	0	0	1	0	0	0	-1	0	0	0	0
15	0.486306	5884.926846583	0.520007179	1.0677	0	0	2	-2	0	0	0	0	0	0	0
16	0.468597	6244.942814354	5.866398759	1.0061	0	0	1	0	0	0	0	-1	0	0	0
17	0.447061	26.298319800	3.615796498	238.9196	0	8	-13	0	0	0	0	0	0	0	0
18	0.435206	-398.149003408	4.349338347	15.7810	0	0	1	-2	0	0	0	0	0	0	0
19	0.432392	74.781598567	2.435898309	84.0205	0	0	0	0	0	0	1	0	0	0	0
20	0.375510	5507.553238667	4.103476804	1.1408	0	3	-4	0	0	0	0	0	0	0	0
21	0.243085	-775.522611324	3.651837925	8.1019	0	3	-5	0	0	0	0	0	0	0	0
22	0.230685	5856.477659115	4.773852582	1.0729	0	0	1	0	0	-2	0	0	0	0	0
23	0.203747	12036.460734888	4.333987818	0.5220	0	0	2	0	-1	0	0	0	0	0	0
24	0.173435	18849.227549974	6.153743485	0.3333	0	0	3	0	0	0	0	0	0	0	0
25	0.159080	10977.078804699	1.890075226	0.5724	0	0	2	0	-3	0	0	0	0	0	0
26	0.143935	-796.298006816	5.957517795	7.8905	0	0	2	-4	0	0	0	0	0	0	0
27	0.137927	11790.629088659	1.135934669	0.5329	0	3	-3	0	0	0	0	0	0	0	0
28	0.119979	38.133035638	4.551585768	164.7701	0	0	0	0	0	0	0	1	0	0	0
29	0.118971	5486.777843175	1.914547226	1.1452	0	0	3	-4	0	0	0	0	0	0	0
30	0.116120	1059.381930189	0.873504123	5.9310	0	0	0	0	2	0	0	0	0	0	0
31	0.101868	-5573.142801634	5.984503847	1.1274	0	0	0	0	0	0	0	0	1	0	-1
32	0.098358	2544.314419883	0.092793886	2.4695	0	0	2	-3	0	0	0	0	0	0	0
33	0.080164	206.185548437	2.095377709	30.4735	0	0	0	0	2	-4	0	0	0	0	0
34	0.079645	4694.002954708	2.949233637	1.3386	0	0	1	0	-3	0	0	0	0	0	0
35	0.075019	2942.463423292	4.980931759	2.1353	0	0	1	-1	0	0	0	0	0	0	0
36	0.064397	5746.271337896	1.280308748	1.0934	0	0	1	0	1	-5	0	0	0	0	0
37	0.063814	5760.498431898	4.167901731	1.0907	0	0	1	0	-3	5	0	0	0	0	0
38	0.062617	20.775395492	2.654394814	302.4340	0	3	-7	4	0	0	0	0	0	0	0
39	0.058844	426.598190876	4.839650148	14.7286	0	0	0	0	0	2	0	0	0	0	0
40	0.054139	17260.154654690	3.411091093	0.3640	0	0	3	0	-3	0	0	0	0	0	0
41	0.048373	155.420399434	2.251573730	40.4270	0	0	8	-15	0	0	0	0	0	0	0
42	0.048042	2146.165416475	1.495846011	2.9276	0	0	3	-5	0	0	0	0	0	0	0
43	0.046551	-0.980321068	0.921573539	6409.3138	0	0	10	-19	0	3	0	0	0	0	0
44	0.042732	632.783739313	5.720622217	9.9294	0	0	0	0	2	-2	0	0	0	0	0
45	0.042560	161000.685737473	1.270837679	0.0390	0	0	0	0	0	0	0	0	1	0	1
46	0.042411	6275.962302991	2.869567043	1.0012	0	0	1	0	2	-5	0	0	0	0	0
47	0.040759	12352.852604545	3.981496998	0.5086	0	0	2	0	0	-1	0	0	0	0	0

$\approx$1138 yr) show discrepancies of the order of 8 ns. The short period term $3\lambda_2 - 5\lambda_3 + \lambda_5$ (period $\approx$25.56 yr) shows a difference of 13 ns. These discrepancies are not explained since the analytical theories used to provide ϱ_i and v for the planets and the Moon in (1) are VSOP 82 (Bretagnon, 1982) and ELP 2000 (Chapront-Touzé and Chapront, 1983) in both formulae.

4. Discussion

The IAU recommendation that only periodic terms be kept in the TB–TT transformation implies that this transformation be built with a General Theory to provide the positions and velocities of the planets for Eq. (1). In such a theory, all the long period terms

Table 1 (continued)

i	A_i (μs)	ω_{ai} (rd/10^{-3}y)	ϕ_{ai} (rd)	Period (years)	Arguments Me	V	E	M	J	S	U	N	D	F	L
48	0.040480	15720.838784878	2.546610123	0.3997	0	4	-4	0	0	0	0	0	0	0	0
49	0.040184	-7.113547001	3.565975565	883.2704	0	0	0	0	2	-5	0	0	0	0	0
50	0.036955	3154.687084896	5.071801441	1.9917	0	4	-6	0	0	0	0	0	0	0	0
51	0.036564	5088.628839767	3.324679049	1.2348	0	0	4	-6	0	0	0	0	0	0	0
52	0.036507	801.820931124	6.248866009	7.8361	0	5	-8	0	0	0	0	0	0	0	0
53	0.034867	522.577418094	5.210064075	12.0235	0	0	0	0	3	-5	0	0	0	0	0
54	0.033529	9437.762934887	2.404714239	0.6657	0	4	-5	0	0	0	0	0	0	0	0
55	0.033477	6062.663207553	4.144987272	1.0364	0	0	1	0	2	-6	0	0	0	0	0
56	0.032438	6076.890301554	0.749317412	1.0339	0	0	1	0	-2	4	0	0	0	0	0
57	0.032423	8827.390269875	5.541473556	0.7118	0	0	3	-3	0	0	0	0	0	0	0
58	0.030215	7084.896781115	3.389610345	0.8868	0	5	-7	0	0	0	0	0	0	0	0
59	0.029862	12139.553509107	1.770181024	0.5176	0	0	2	0	0	-2	0	0	0	0	0
60	0.029247	-71430.695617928	4.183178762	0.0880	0	0	1	0	0	0	0	0	-1	0	0
61	0.028244	-6286.598968340	5.069663519	0.9995	0	0	3	-8	3	0	0	0	0	0	0
62	0.027567	6279.552731642	5.040846034	1.0006	0	0	5	-8	3	0	0	0	0	0	0
63	0.025196	1748.016413067	2.901883301	3.5945	0	0	4	-7	0	0	0	0	0	0	0
64	0.024816	-1194.447010225	1.087136918	5.2603	0	0	3	-6	0	0	0	0	0	0	0
65	0.022567	6133.512652857	3.307984806	1.0244	0	0	1	0	0	0	-2	0	0	0	0
66	0.022509	10447.387839604	1.460726241	0.6014	0	0	2	0	-4	0	0	0	0	0	0
67	0.021691	14143.495242431	5.952658009	0.4442	0	2	-1	0	0	0	0	0	0	0	0
68	0.020937	8429.241266467	0.652303414	0.7454	0	0	4	-5	0	0	0	0	0	0	0
69	0.020322	419.484643875	3.735430632	14.9783	0	0	0	0	2	-3	0	0	0	0	0
70	0.017673	6812.766815086	3.186129845	0.9223	0	0	1	0	1	0	0	0	0	0	0
71	0.017806	73.297125859	3.475975097	85.7221	0	0	0	0	0	0	2	-2	0	0	0
72	0.016155	10213.285546211	1.331103168	0.6152	0	1	0	0	0	0	0	0	0	0	0
73	0.015974	-2352.866153772	6.145309371	2.6704	0	1	-2	0	0	0	0	0	0	0	0
74	0.015949	-220.412642439	4.005298270	28.5065	0	0	0	0	2	-6	0	0	0	0	0
75	0.015078	19651.048481098	3.969480770	0.3197	0	5	-5	0	0	0	0	0	0	0	0
76	0.014751	1349.867409659	4.308933301	4.6547	0	0	5	-9	0	0	0	0	0	0	0
77	0.014318	16730.463689596	3.016058075	0.3756	0	0	3	0	-4	0	0	0	0	0	0
78	0.014223	17789.845619785	2.104551349	0.3532	0	0	3	0	-2	0	0	0	0	0	0
79	0.013671	-536.804512095	5.971672571	11.7048	0	0	0	0	1	-5	0	0	0	0	0
80	0.012462	103.092774219	1.737438797	60.9469	0	0	0	0	1	-2	0	0	0	0	0
81	0.012420	4690.479836359	4.734090399	1.3396	0	0	5	-8	0	0	0	0	0	0	0
82	0.011942	8031.092263058	2.053414715	0.7824	0	0	5	-7	0	0	0	0	0	0	0
83	0.011847	5643.178563677	5.489005403	1.1134	0	0	1	0	0	-3	0	0	0	0	0
84	0.011707	-4705.732307544	2.654125618	1.3352	0	2	-4	0	0	0	0	0	0	0	0
85	0.011622	5120.601145584	4.863931876	1.2270	0	0	1	0	-3	2	0	0	0	0	0
86	0.010962	3.590428652	2.196567739	1749.9819	0	0	4	-8	1	5	0	0	0	0	0
87	0.010825	553.569402842	0.842715011	11.3503	0	0	7	-13	0	0	0	0	0	0	0
88	0.010396	951.718406251	5.717799605	6.6019	0	0	6	-11	0	0	0	0	0	0	0
89	0.010453	5863.591206116	1.913704550	1.0716	0	0	1	0	-2	3	0	0	0	0	0
90	0.010099	283.859318865	1.942176992	22.1349	0	3	-5	0	2	0	0	0	0	0	0
91	0.009858	6309.374169791	1.061816410	0.9958	0	8	-12	0	0	0	0	0	0	0	0
92	0.009963	149.563197135	4.870690598	42.0102	0	0	0	0	0	0	2	0	0	0	0
93	0.009370	149854.400134205	0.673880395	0.0419	0	0	0	0	0	0	0	0	3	0	-1

such as the motions of the nodes and perihelie of the planets (with periods ranging from 45000 to 2000000 yr), are kept as sine terms and are not expanded relative to time. Such a theory, which is valid over millions of years, is not precise locally in time and would not provide the 1 ns level of accuracy sought in the TB–TT transformation. Even is such a precise General Theory were available, it would probably not be worthwhile using it for the TB–TT transformation. Such a transformation would be precise and valid over millions of years but would be complicated and unwieldy with many more terms than the 750 we obtain for our 1 ns formula. Practically, a transformation from TB to TT is only needed for a few thousand years and secular variation theories are ideal for this purpose. They provide accurate positions of the planets by construction because the long period terms are expanded as time polynomials. For example, the 93462 yr term that is kept by Hirayama et al., is developed with respect to time in our formula because its period is a 100 times the range of use of our formula. Hence, although the IAU recommendation suggests that

Table 1 (continued)

i	B_i (μs)	ω_{bi} (rd/10^{-3}y)	ϕ_{bi} (rd)	Period (years)	Me	V	E	M	J	S	U	N	D	F	L
1	102.156724	6283.075849991	4.249032005	1.0000	0	0	1	0	0	0	0	0	0	0	0
2	1.706807	12566.151699983	4.205904248	0.5000	0	0	2	0	0	0	0	0	0	0	0
3	0.269668	213.299095438	3.400290479	29.4572	0	0	0	0	0	1	0	0	0	0	0
4	0.265919	529.690965095	5.836047367	11.8620	0	0	0	0	1	0	0	0	0	0	0
5	0.210568	-3.523118349	6.262738348	1783.4159	0	0	4	-8	3	0	0	0	0	0	0
6	0.077996	5223.693919802	4.670344204	1.2028	0	0	1	0	-2	0	0	0	0	0	0
7	0.059146	26.298319800	1.083044735	238.9196	0	8	-13	0	0	0	0	0	0	0	0
8	0.054764	1577.343542448	4.534800170	3.9834	0	2	-3	0	0	0	0	0	0	0	0
9	0.034420	-398.149003408	5.980077351	15.7810	0	0	1	-2	0	0	0	0	0	0	0
10	0.033595	5507.553238667	5.980162321	1.1408	0	3	-4	0	0	0	0	0	0	0	0
11	0.032088	18849.227549974	4.162913471	0.3333	0	0	3	0	0	0	0	0	0	0	0
12	0.029198	5856.477659115	0.623811863	1.0729	0	0	1	0	0	-2	0	0	0	0	0
13	0.027764	155.420399434	3.745318113	40.4270	0	0	8	-15	0	0	0	0	0	0	0
14	0.025190	5746.271337896	2.980330535	1.0934	0	0	1	0	1	-5	0	0	0	0	0
15	0.024976	5760.498431898	2.467913690	1.0907	0	0	1	0	-3	5	0	0	0	0	0
16	0.022997	-796.298006816	1.174411803	7.8905	0	0	2	-4	0	0	0	0	0	0	0
17	0.021774	206.185548437	3.854787540	30.4735	0	0	0	0	2	-4	0	0	0	0	0
18	0.017925	-775.522611324	1.092065955	8.1019	0	3	-5	0	0	0	0	0	0	0	0
19	0.013794	426.598190876	2.699831988	14.7286	0	0	0	0	0	2	0	0	0	0	0
20	0.013276	6062.663207553	5.845801920	1.0364	0	0	1	0	2	-6	0	0	0	0	0
21	0.012869	6076.890301554	5.333425680	1.0339	0	0	1	0	-2	4	0	0	0	0	0
22	0.012152	1059.381930189	6.222874454	5.9310	0	0	0	0	2	0	0	0	0	0	0
23	0.011774	12036.460734888	2.292832062	0.5220	0	0	2	0	-1	0	0	0	0	0	0
24	0.011081	-7.113547001	5.154724984	883.2704	0	0	0	0	2	-5	0	0	0	0	0
25	0.010143	4694.002954708	4.044013795	1.3386	0	0	1	0	-3	0	0	0	0	0	0
26	0.010084	522.577418094	0.749320262	12.0235	0	0	0	0	3	-5	0	0	0	0	0
27	0.009357	5486.777843175	3.416081409	1.1452	0	0	3	-4	0	0	0	0	0	0	0

i	C_i (μs)	ω_{ci} (rd/10^{-3}y)	ϕ_{ci} (rd)	Period (years)	Me	V	E	M	J	S	U	N	D	F	L
0	0.370115	0.000000000	4.712388980	—	0	0	0	0	0	0	0	0	0	0	0
1	4.322990	6283.075849991	2.642893748	1.0000	0	0	1	0	0	0	0	0	0	0	0
2	0.122605	12566.151699983	2.438140634	0.5000	0	0	2	0	0	0	0	0	0	0	0
3	0.019476	213.299095438	1.642186981	29.4572	0	0	0	0	0	1	0	0	0	0	0
4	0.016916	529.690965095	4.510959344	11.8620	0	0	0	0	1	0	0	0	0	0	0
5	0.013374	-3.523118349	1.502210314	1783.4159	0	0	4	-8	3	0	0	0	0	0	0

i	D_i (μs)	ω_{di} (rd/10^{-3}y)	ϕ_{di} (rd)	Period (years)	Me	V	E	M	J	S	U	N	D	F	L
1	0.143388	6283.075849991	1.131453581	1.0000	0	0	1	0	0	0	0	0	0	0	0

General Theories be used, only secular variation theories and numerical integrations meet the precision and practicality requirements.

The TB–TT transformation is used in pulsar timing analysis to transform the observed arrival times of pulses at the observatory to a uniform standard of time TB. The pulsar "time", provided by the observation of its pulses, can thus be compared to a uniform solar system time scale. The transformed barycentric time will also provide the time argument of the ephemeris used for the position of the Earth, needed in the analysis. Besides the TB–TT transformation, the transformation from a measured arrival time to a barycentric time includes corrections between the observatory clock and a national standard atomic clock and a diurnal term accounting for the observatory position with respect to the geocentre. These two corrections to a measured arrival time will give a time in TT. Different national and international standard atomic time scales are available (from the USNO, the NBS, the PTB, the BIPM,...) and the transformation from measured arrival times to TT and thus to barycentric times TB can be constructed with these different atomic time scales. Therefore not only should realizations of TT be identified with the atomic time scale used, as suggested by Guinot and Seidelmann and recommended by the Working group on Reference Frames at the IAU XX General Assembly, but TB should also be identified in the same manner. TB will of course also depend on the time transformation used, be it numerical or analytical. The transformation used should be made clear in the identification of TB. For example, TT(TAI) and TB(TAI, χ) would represent respectively, the realization of TT obtained by using the atomic time scale TAI and the barycentric time obtained by substituting TT(TAI) in the TB–TT transformation denoted by the symbol χ. This is important for pulsar timing analysis. We have shown (Fairhead, 1989)

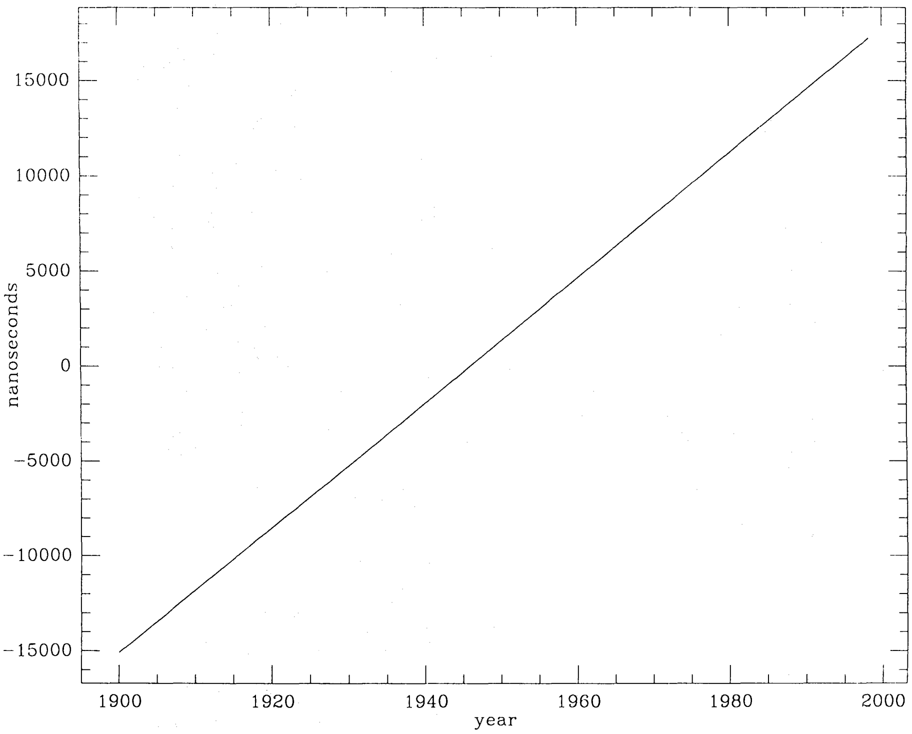

Fig. 1a. Difference between the analytical formula and the JPL numerical transformation

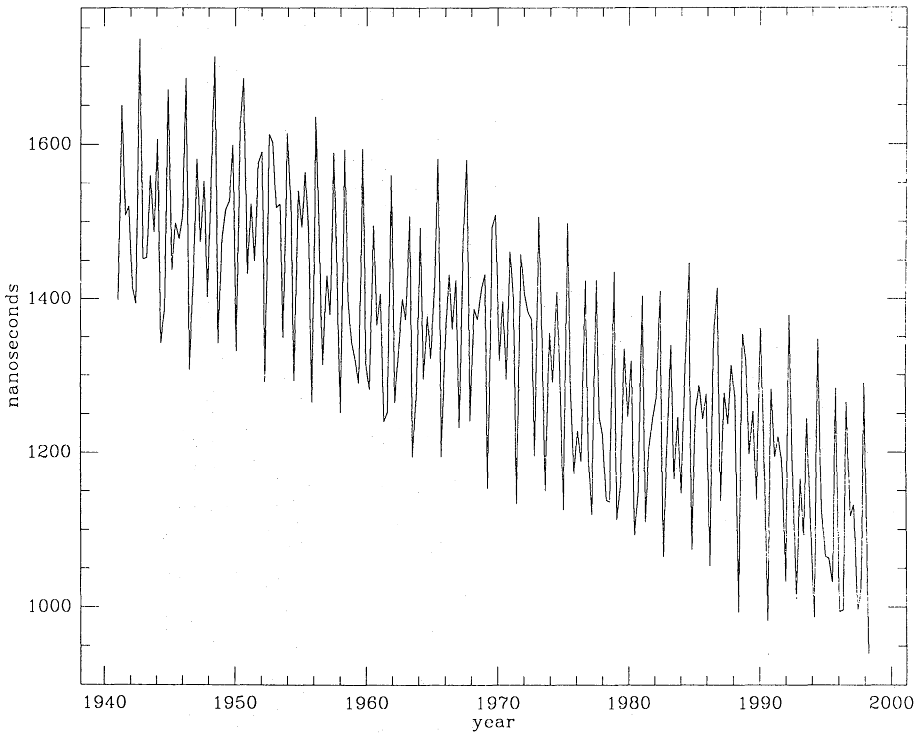

Fig. 1b. Difference between the analytical formula and the CfA numerical transformation

that the use of different atomic time scales in the realization of TT and TB leads to systematic astrometric errors in the parameters determined for the millisecond pulsar PSR 1937+214 (difference in the periods of $0.73\ 10^{-15}$ s when comparing results obtained with TB(USNO, BDL) and TB(BIPM, BDL), where BDL stands for our analytic TB–TT transformation; this is due to the different definitions of these two time scales). These errors are larger than the formal uncertainties calculated (14σ) and are also larger than the discrepancies introduced in the period when using the numerical TT–TB formula from the JPL or our analytical formula.

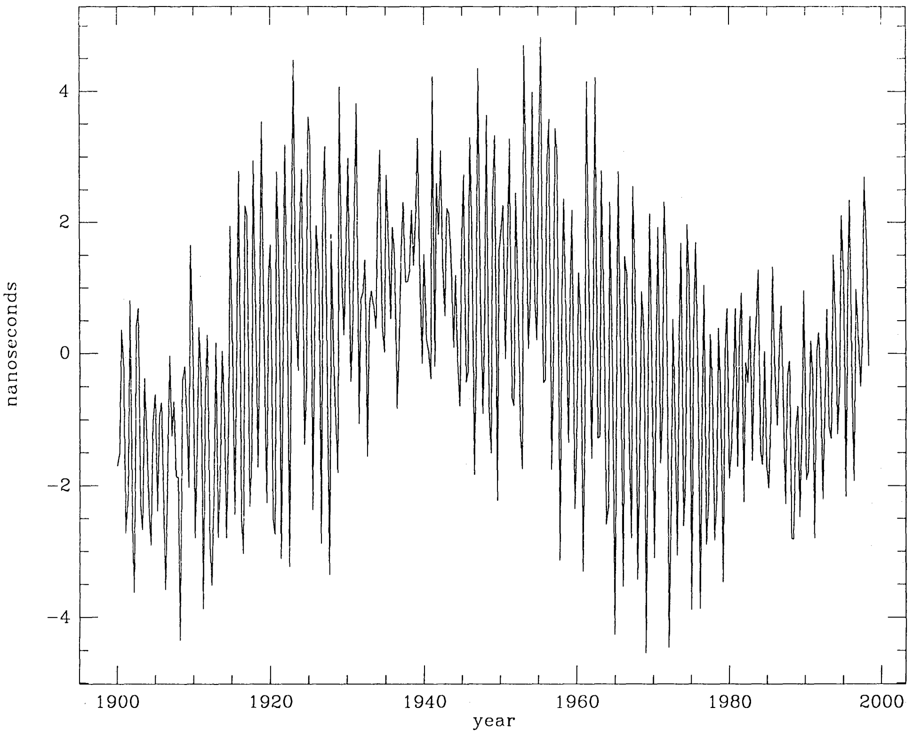

Fig. 2. Differences between the JPL transformation and our formula when the linear trend is removed. The differences were calculated every 100 days. The time interval is from 1900 to 2000. The masses used in the ephemeris providing the position of the planets in our formula are those recommended by the IAU

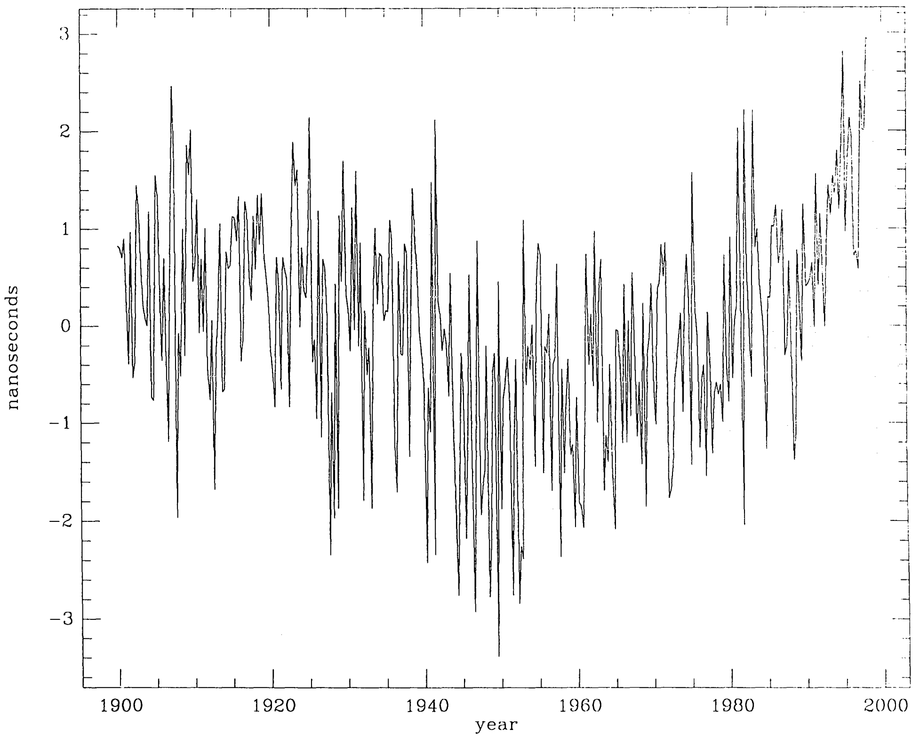

Fig. 3. Same as Fig. 2 but the masses used in our formula are those provided by the JPL

5. Conclusion

For the time transformation TB–TT, numerical "time ephemeris" integrated over various time intervals exhibit mutual linear drifts. The magnitude of this drift depends on the length of the time interval and on its boundaries. This difficulty vanishes when one uses an analytical expression of this time transformation even if one considers an interval as long as a few thousand years around J 2000. The expression (3) above and the coefficients given in Table 1 provide a time transformation TB–TT accurate at the 100 ns level. However, we have computed a complete expression accurate at the 1 ns level which is available on request.

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