GammaLib - Feature #1017

Improve GWcs.solidangle method

12/04/2013 11:13 AM - Deil Christoph

Status:	Closed	**Start date:**	12/04/2013
Priority:	High	**Due date:**	
Assigned To:	Owen Ellis	**% Done:**	100%
Category:		**Estimated time:**	0.00 hour
Target version:	00-08-00		

Description

The docstring of the `GWcs.omega` method to compute the pixel solid angle mentions "Something more intelligent should be implemented in the future."
http://gammalib.sourceforge.net/doxygen/classGWcs.html#e2c412e6435e058fb80c04b958f47f79

I started a diffuse emission analysis for large parts of the sky and am using AIT images.
I thought in AIT images the pixel area is almost constant, but I find that using the current method the pixel area varies by a factor 2.
See https://github.com/cdeil/gammalib-tutorials/blob/master/make_omega_image.py and attached screenshot.

Is this an error in the omega calculation for the current method or are AIT pixels not really equal-area?

I've looked around a bit, and I'd be happy to implement a more precise method, but I'm not sure which is best.

http://en.wikipedia.org/wiki/Spherical_trigonometry#Area_and_spherical_excess
http://www.mathworks.de/de/help/map/ref/areaint.html
http://stackoverflow.com/questions/4681737/how-to-calculate-the-area-of-a-polygon-on-the-earths-surface-using-python

History

#1 - 12/04/2013 03:06 PM - Knödlseder Jürgen

Deil Christoph wrote:

> The docstring of the `GWcs.omega` method to compute the pixel solid angle mentions "Something more intelligent should be implemented in the future."
> http://gammalib.sourceforge.net/doxygen/classGWcs.html#e2c412e6435e058fb80c04b958f47f79

Right. I was looking for some general formulae for solid angles of pixels for various WCS projections, yet I did not find anything. So the dumb method just takes the corners of a pixel to estimate the solid angle, which probably isn't very good if the pixels are considerably distorted.

If you have an explicit formula for some of the projections one can add this to the projection class by overloading the

```
virtual double omega(const GSkyPixel& pixel) const;
```

method of the GWcs base class.

#2 - 12/05/2013 03:11 PM - Deil Christoph

I've asked if someone knows a good method on the astropy mailing list:
http://mail.scipy.org/pipermail/astropy/2013-December/002937.html

#3 - 12/05/2013 08:40 PM - Knödlseder Jürgen

- Priority changed from Normal to High

Back in 2009, I asked the question already to Mark Calabretta (Mister WCS). He was not aware of some software doing this job:

> managed to get a general conversion. Do you know about a collection
> of formula that can be used to derive the solid angles of individual
> image pixels or are there some subroutines in the wcslib to do this
> task (or are there some subroutines available for idl)?

> Dear Jürgen,
>
> It's difficult in general to evaluate pixel areas analytically - a
> square pixel in the plane of projection may become severely distorted
> when projected back onto the sphere. Cylindrical projections and equal
> area projections are really the only easy cases, numerical methods must
> be used for the others. However, I'm not aware of any software that
> can do this.
>
> Regards,
> Mark Calabretta

I put the priority of this issue to **High** as we should really not forget about this. Maybe we need a maths geek who works out the formula, or who can propose a solid numerical method.

#4 - 12/06/2013 04:30 AM - Deil Christoph

It seems the usual algorithm is to project the pixel corners on the sphere and then using Girard's theorem:
http://mail.scipy.org/pipermail/astropy/2013-December/002941.html
http://montage.ipac.caltech.edu/docs/algorithms.html

There is a stand-alone C version of the core Montage algorithm:
http://mail.scipy.org/pipermail/astropy/2013-December/002940.html
https://github.com/astrofrog/python-reprojection/pull/3

I think Montage has a non-free license, but maybe this part will become available as open source:

http://montage.ipac.caltech.edu/docs/download.html
https://github.com/astrofrog/python-reprojection/issues/1

If not we can always re-implement it (or the sub-set of solid angle computation we need), but presumably there's some corner cases to get right and the existing implementation is well-tested.

#5 - 12/06/2013 11:23 AM - Deil Christoph

- Assigned To set to Deil Christoph

https://github.com/astrofrog/python-reprojection/blob/master/reproject/overlapArea.c
is now under a BSD license, i.e. I think we can include a copy in gammalib.

Tom and I are currently improving it and once it's in good shape (no global variables, good test coverage) in a week or two we can put a copy in gammalib:
https://github.com/astrofrog/python-reprojection/issues

The core functionality implemented there is flux-conserving reprojection, not only computation of pixel solid angles.
I think we want flux-conserving reprojection methods in gammalib as well, e.g. it could come in handy to stack runs into survey maps or to reproject background models in the FOV system onto sky maps.

Jürgen, do you agree?
If yes, can you please propose a place and outline what API you think would be nice (class method? function?)

#6 - 12/10/2013 12:12 PM - Knödlseder Jürgen

Note that GWcs.omega() has been renamed to GWcs.solidangle()

#7 - 01/13/2014 09:17 PM - Deil Christoph

- Subject changed from Improve GWcs.omega method? to Improve GWcs.solidangle method

- Assigned To changed from Deil Christoph to Owen Ellis

- Target version set to 2nd coding sprint

Ellis, could you please implement the method to compute the solid angle of a given pixel described here?
http://mail.scipy.org/pipermail/astropy/2013-December/002940.html

#8 - 01/17/2014 01:43 PM - Owen Ellis

- Status changed from New to In Progress

#9 - 01/17/2014 03:27 PM - Owen Ellis

- File image_issue.png added

I replaced the existing solid angle script the following:

```
{
   GSkyDir dir1 = pix2dir(GSkyPixel(pixel.x()-0.5, pixel.y()-0.5));
   GSkyDir dir2 = pix2dir(GSkyPixel(pixel.x()+0.5, pixel.y()-0.5));
   GSkyDir dir3 = pix2dir(GSkyPixel(pixel.x()+0.5, pixel.y()+0.5));
```

```
    GSkyDir dir4 = pix2dir(GSkyPixel(pixel.x()-0.5, pixel.y()+0.5));

    GVector vec1 = dir1.celvector();
    GVector vec2 = dir2.celvector();
    GVector vec3 = dir3.celvector();
    GVector vec4 = dir4.celvector();

    double angle1 = std::acos(cross(vec2, (cross(vec1, vec2))) * cross(vec2, (cross(vec3, vec2))));
    double angle2 = std::acos(cross(vec3, (cross(vec2, vec3))) * cross(vec3, (cross(vec4, vec3))));
    double angle3 = std::acos(cross(vec4, (cross(vec3, vec4))) * cross(vec4, (cross(vec1, vec4))));
    double angle4 = std::acos(cross(vec1, (cross(vec4, vec1))) * cross(vec1, (cross(vec2, vec1))));

    double solidangle = (angle1 + angle2 + angle3 + angle4) - (2 * gammalib::pi);

    return solidangle;
}
```

Which gives clearly incorrect results: I would expect the map to have pixel values of approx 1 but they are around 1e-4 (and quite variable) - see attached ds9 screenshot - the test script is here: https://gist.github.com/ellisowen/8473696

I shall start de-bugging, but wondered if you could see anything obviously wrong with the method/script?

Also, would it be best to include this as an alternative to the original solidangle calculation, or as a replacement?

(Branch is here: https://github.com/ellisowen/gammalib/compare/issue_1017)

#10 - 01/17/2014 04:50 PM - Knödlseder Jürgen

1e-4 looks like an issue with square degrees versus steradians. Note that the solid angle is computed in steradians, hence it's probably not so bad. 1 square degree is about 3.05e-4 steradians. All your angles above are in radians.

I guess that for an Aitoff projection, things become really complex when it comes to the pole. Maybe the formula you use can not manage this?

A minor thing, not related to the problems you have. I see

```
double solidangle = (angle1 + angle2 + angle3 + angle4) - (2 * gammalib::pi);
```

in your code. Please always use

```
double solidangle = (angle1 + angle2 + angle3 + angle4) - (2.0 * gammalib::pi);
```

hence floating points when you mean floating points. I had some issue with such things in the past, I think on FreeBSD (don't remember exactly).

#11 - 01/17/2014 05:51 PM - Knödlseder Jürgen

- File aitoff-solid-angle.png added

I got the following result with this code:

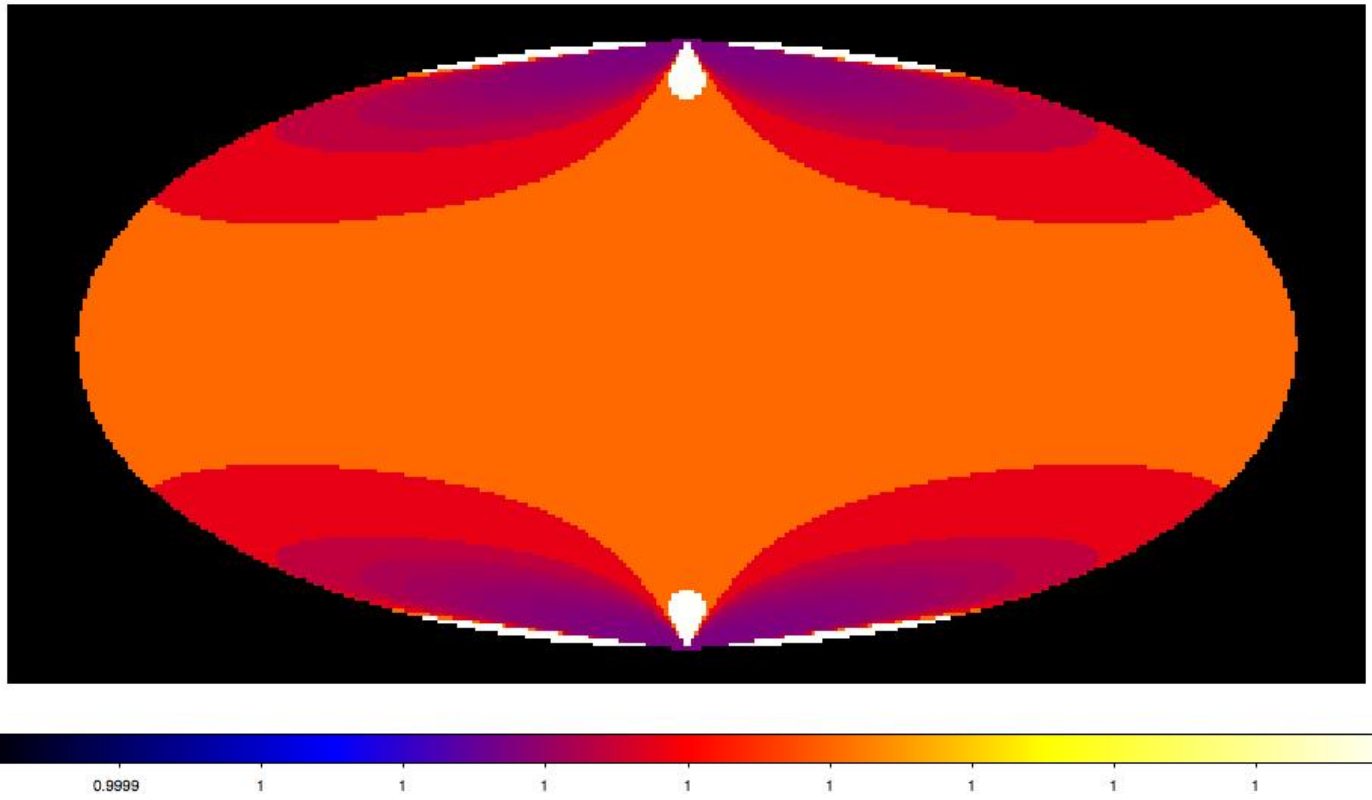
```
//
double a;
double b;
double c;

// Angle 1
a = dir1.dist(dir3);
b = dir4.dist(dir3);
c = dir4.dist(dir1);
double angle1 = std::acos((std::cos(a) - std::cos(b)*std::cos(c))/(std::sin(b)*std::sin(c)));

// Angle 2
a = dir4.dist(dir2);
b = dir2.dist(dir3);
c = dir3.dist(dir4);
double angle2 = std::acos((std::cos(a) - std::cos(b)*std::cos(c))/(std::sin(b)*std::sin(c)));

// Angle 3
a = dir1.dist(dir3);
b = dir1.dist(dir2);
c = dir2.dist(dir3);
double angle3 = std::acos((std::cos(a) - std::cos(b)*std::cos(c))/(std::sin(b)*std::sin(c)));

// Angle 4
a = dir4.dist(dir2);
b = dir4.dist(dir1);
c = dir1.dist(dir2);
double angle4 = std::acos((std::cos(a) - std::cos(b)*std::cos(c))/(std::sin(b)*std::sin(c)));
```



Looks reasonable. The code uses spherical triangles to compute the inner angles of the Polygon. I guess you code attempts to do the same thing, but maybe some stuff got mixed up.

My code is not yet optimized as I repeat distance computations. But I just wanted to see whether this works.

#12 - 01/17/2014 06:01 PM - Knödlseder Jürgen

Here some optimized code that gives the same result.

```
// Compute angular distances between pixel corners
double a12 = dir1.dist(dir2);
double a13 = dir1.dist(dir3);
double a14 = dir1.dist(dir4);
double a23 = dir2.dist(dir3);
double a24 = dir2.dist(dir4);
double a34 = dir3.dist(dir4);

// Angle 1
double a = a13;
double b = a34;
double c = a14;
double angle1 = std::acos((std::cos(a) - std::cos(b)*std::cos(c))/(std::sin(b)*std::sin(c)));

// Angle 2
a = a24;
b = a23;
c = a34;
double angle2 = std::acos((std::cos(a) - std::cos(b)*std::cos(c))/(std::sin(b)*std::sin(c)));

// Angle 3
a = a13;
b = a12;
c = a23;
double angle3 = std::acos((std::cos(a) - std::cos(b)*std::cos(c))/(std::sin(b)*std::sin(c)));

// Angle 4
a = a24;
b = a14;
c = a12;
double angle4 = std::acos((std::cos(a) - std::cos(b)*std::cos(c))/(std::sin(b)*std::sin(c)));
```

I pushed this into issue_1017_jk. We should test all projections before we decide to merge this in to make sure that the computations are okay.

#13 - 01/17/2014 06:02 PM - Knödlseder Jürgen

- % Done changed from 0 to 50

Just think: can still be optimized a little more as there are repeated cos and sin calls.

#14 - 01/17/2014 06:09 PM - Owen Ellis

- % Done changed from 50 to 0

Okay, I think I also have figured out the problem with my method. I avoid calling cos and sin so much, so maybe it would need less optimizing?

I'll push it to https://github.com/ellisowen/gammalib/compare/issue_1017 when I've done a couple of checks!

#15 - 01/17/2014 06:10 PM - Owen Ellis

- % Done changed from 0 to 50

(Changed % done by accident so just returned it to 50%)

#16 - 01/17/2014 06:49 PM - Owen Ellis

- File new_aitoff.png added

- Status changed from In Progress to Pull request

Pushed to https://github.com/ellisowen/gammalib/compare/issue_1017 - please review.

Attached check projection ds9 screenshot (it's not very clear, but values seem reasonable).

#17 - 01/17/2014 09:29 PM - Knödlseder Jürgen

- Status changed from Pull request to In Progress

Sorry to say that there is still some problem.

I tried your code with a pixel size of (0.5 x 0.5) instead of (1.0 x 1.0), and when you integrate over the map you get a total solid angle that is 4 times too large. With the trigonometric formula the correct values are obtained.

For the (1.0 x 1.0) pixel size I still get an error of a few percent when being close to the pole, so the formula seem only to be approximate.

Can you point to where you took the cross-product / dot-product formula from? Could it be that they only work for cartesian coordinates?

#18 - 01/17/2014 09:41 PM - Knödlseder Jürgen

Addition: I checked for the CAR projection, and here the formula obviously does not work at all (there is basically no variation over the sky, while for a CAR projection the pixels become smaller when going to the poles).

#19 - 01/17/2014 11:22 PM - Knödlseder Jürgen

- File make_solidangle_image.py added

- File solidangle_AIT.png added

- File solidangle_AZP.png added

- File solidangle_CAR.png added

- File solidangle_MER.png added

- File solidangle_STG.png added

- File solidangle_TAN.png added

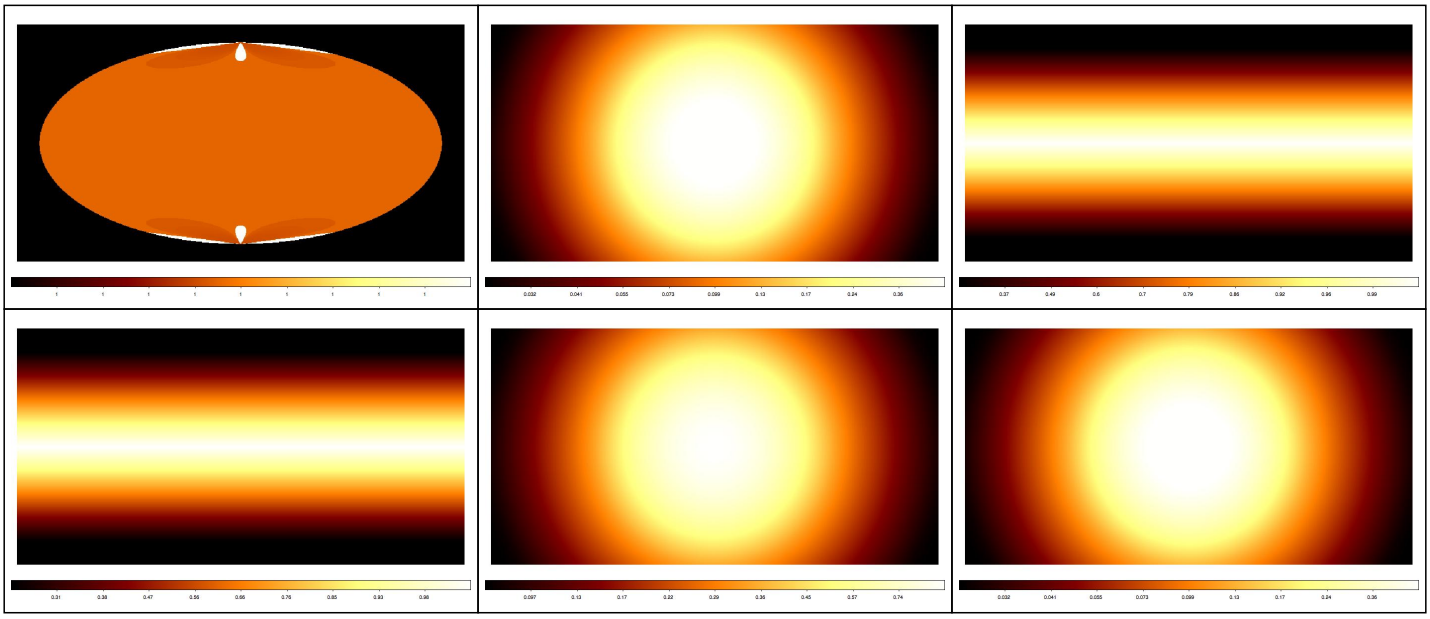
- Status changed from In Progress to Feedback

- % Done changed from 50 to 100

I modified the code so that both methods can be toggled using a compiler flag. This allows you to investigate this further. Pushed into issue_1017_jk.

I optimized the code a little more for the trigonometric version, and also take care of the possibility that a pixel touches the pole, reducing the polygon to a triangle. I tested the code with the script attached.

Below also a number of all sky maps produced by the script for various projections. They seem to be all okay (at least visually). The script now also computes the total solid angle of the map, and at least non of the projections led to a NaN. AIT and CAR give 4 pi as expected. Eventually, some more testing would be nice.



#20 - 01/17/2014 11:45 PM - Deil Christoph

Knödlseder Jürgen wrote:

> Can you point to where you took the cross-product / dot-product formula from? Could it be that they only work for cartesian coordinates?

I asked on the astropy mailing list how to best compute this and this is a formula from Michael Droettboom gave:
http://mail.scipy.org/pipermail/astropy/2013-December/002940.html

I didn't think about it or look at your or Ellis's code in detail.
If your implementation with trigonometric functions looks OK we should simply use that, I guess.
One more thing that could be worth testing is very small pixels (say 0.1, 0.01, 0.001 deg size) to make sure that rounding errors with doubles will be acceptable for CTA.

#21 - 01/17/2014 11:57 PM - Owen Ellis

The factor of 4 issue you mentioned I think is because I just noticed I hadn't adapted the term ((gammalib::pi * (1.0/180.0)) in the solid angle formula to account for different pixel sizes.

However, I'd tend to agree with Christoph - the formula in my code clearly isn't quite right (even with the above problem corrected), and it's not yet clear to me how to fix the other issues. So I think it would perhaps better to just use your working code?

#22 - 01/18/2014 02:40 AM - Knödlseder Jürgen

Deil Christoph wrote:

> Knödlseder Jürgen wrote:
>
> > Can you point to where you took the cross-product / dot-product formula from? Could it be that they only work for cartesian coordinates?
>
> I asked on the astropy mailing list how to best compute this and this is a formula from Michael Droettboom gave:
> http://mail.scipy.org/pipermail/astropy/2013-December/002940.html
>
> I didn't think about it or look at your or Ellis's code in detail.
> If your implementation with trigonometric functions looks OK we should simply use that, I guess.
> One more thing that could be worth testing is very small pixels (say 0.1, 0.01, 0.001 deg size) to make sure that rounding errors with doubles will be acceptable for CTA.

I check with very small pixels (0.0001 deg) and it's okay. I see numerical noise at the 1e-4 level.

But you're right, my initial trigonometric code has numerical problems. I finally managed to solve them by dividing internally the pixel into two triangles and using Huilier's theorem (see http://mathworld.wolfram.com/SphericalExcess.html). This also needs a little less trigonometric functions, though it requires tangens instead of sine or cosine, which is a bit slower (see [[Computation Benchmarks]]).

#23 - 01/18/2014 02:42 AM - Knödlseder Jürgen

- Target version changed from 2nd coding sprint to 00-08-00

Owen Ellis wrote:

> The factor of 4 issue you mentioned I think is because I just noticed I hadn't adapted the term ((gammalib::pi * (1.0/180.0)) in the solid angle formula to account for different pixel sizes.
>
> However, I'd tend to agree with Christoph - the formula in my code clearly isn't quite right (even with the above problem corrected), and it's not yet clear to me how to fix the other issues. So I think it would perhaps better to just use your working code?

For the moment I left all options in the code, can be selected through #define. In the long run we may remove the obsolete code, but we better keep for the moment the possibility to play with the options.

I now merged all the stuff in devel.

#24 - 01/19/2014 02:01 AM - Knödlseder Jürgen

- Status changed from Feedback to Closed

Files

Screen Shot 2013-12-04 at 10.05.39.png	139 KB	12/04/2013	Deil Christoph
image_issue.png	13.6 KB	01/17/2014	Owen Ellis
aitoff-solid-angle.png	23.8 KB	01/17/2014	Knödlseder Jürgen
new_aitoff.png	10.2 KB	01/17/2014	Owen Ellis
make_solidangle_image.py	867 Bytes	01/17/2014	Knödlseder Jürgen
solidangle_AIT.png	16.6 KB	01/17/2014	Knödlseder Jürgen
solidangle_AZP.png	157 KB	01/17/2014	Knödlseder Jürgen
solidangle_CAR.png	8.58 KB	01/17/2014	Knödlseder Jürgen
solidangle_MER.png	8.71 KB	01/17/2014	Knödlseder Jürgen
solidangle_STG.png	158 KB	01/17/2014	Knödlseder Jürgen
solidangle_TAN.png	157 KB	01/17/2014	Knödlseder Jürgen