

## GammaLib - Bug #1299

### Fitting problems with radial disk model

07/25/2014 10:17 PM - Knödlseider Jürgen

<b>Status:</b>	Closed	<b>Start date:</b>	07/25/2014
<b>Priority:</b>	Normal	<b>Due date:</b>	
<b>Assigned To:</b>	Knödlseider Jürgen	<b>% Done:</b>	100%
<b>Category:</b>		<b>Estimated time:</b>	0.00 hour
<b>Target version:</b>	00-09-00		
<b>Description</b>			
<p>While finalizing the stacked cube analysis (#1291) I encountered a problem with fitting the radial disk model. For stacked cube analysis, the fit converges badly. But even for unbinned analysis, the fit takes many iterations. Here for example a log for unbinned fitting:</p> <pre>&gt;Iteration 0: -logL=34182.372, Lambda=1.0e-03 &gt;Iteration 1: -logL=34180.081, Lambda=1.0e-03, delta=2.290, max( grad )=19.555723 [DEC:1] &gt;Iteration 2: -logL=34180.070, Lambda=1.0e-04, delta=0.011, max( grad )=-6.847438 [Radius:2] &gt;Iteration 3: -logL=34180.070, Lambda=1.0e-05, delta=0.001, max( grad )=2.516642 [Radius:2] &gt;Iteration 4: -logL=34180.070, Lambda=1.0e-06, delta=0.000, max( grad )=-3.717756 [Radius:2] Iteration 5: -logL=34180.070, Lambda=1.0e-07, delta=-0.000, max( grad )=3.502109 [Radius:2] (stalled) Iteration 6: -logL=34180.070, Lambda=1.0e-06, delta=-0.000, max( grad )=3.502427 [Radius:2] (stalled) Iteration 7: -logL=34180.070, Lambda=1.0e-05, delta=-0.000, max( grad )=3.501858 [Radius:2] (stalled) Iteration 8: -logL=34180.070, Lambda=1.0e-04, delta=-0.000, max( grad )=3.501880 [Radius:2] (stalled) Iteration 9: -logL=34180.070, Lambda=1.0e-03, delta=-0.000, max( grad )=3.499197 [Radius:2] (stalled) Iteration 10: -logL=34180.070, Lambda=1.0e-02, delta=-0.000, max( grad )=3.455784 [Radius:2] (stalled) Iteration 11: -logL=34180.070, Lambda=1.0e-01, delta=-0.000, max( grad )=2.826601 [Radius:2] (stalled) Iteration 12: -logL=34180.070, Lambda=1.0e+00, delta=-0.000, max( grad )=2.395730 [Radius:2] (stalled) &gt;Iteration 13: -logL=34180.070, Lambda=1.0e+01, delta=0.000, max( grad )=-3.178778 [DEC:1] Iteration 14: -logL=34180.070, Lambda=1.0e+00, delta=-0.000, max( grad )=2.706520 [DEC:1] (stalled) Iteration 15: -logL=34180.070, Lambda=1.0e+01, delta=-0.000, max( grad )=-1.960376 [RA:0] (stalled) &gt;Iteration 16: -logL=34180.070, Lambda=1.0e+02, delta=0.000, max( grad )=-1.937920 [RA:0] === GObservations === Number of observations ....: 1 Number of predicted events : 4038 === GModelSky === Number of spatial par's ....: 3 RA .....: 83.6324 +/- 0.00394119 [-360,360] deg (free,scale=1) DEC .....: 22.0138 +/- 0.00327793 [-90,90] deg (free,scale=1) Radius .....: 0.201159 +/- 0.00305998 [0.01,10] deg (free,scale=1) Number of spectral par's ..: 3 Prefactor .....: 5.45432e-16 +/- 1.99357e-17 [1e-23,1e-13] ph/cm2/s/MeV (free,scale=1e-16,gradient) Index .....: -2.45887 +/- 0.0267077 [-0,-5] (free,scale=-1,gradient) === GCTAModelRadialAcceptance === Number of radial par's ....: 1 Sigma .....: 3.05822 +/- 0.039788 [0.01,10] deg2 (free,scale=1,gradient) Number of spectral par's ..: 3 Prefactor .....: 5.99601e-05 +/- 1.76021e-06 [0,0.001] ph/cm2/s/MeV (free,scale=1e-06,gradient) Index .....: -1.85301 +/- 0.0164761 [-0,-5] (free,scale=-1,gradient)</pre> <p>The fit stalls often (which is also the case for binned and stacked cube analysis) and takes many iterations to terminate. Note that for the above example, the numerical integration step size was set to 1e-5.</p>			
<b>Related issues:</b>			
Related to GammaLib - Action # 1291: Finish implementation of stacked cube an...	Closed	07/23/2014	
Related to GammaLib - Bug # 1259: Many integration warnings shown when fittin...	Closed	07/09/2014	
Related to GammaLib - Bug # 1239: GModelSpatialEllipticalDisk Integration	Closed	07/01/2014	
Related to GammaLib - Action # 1303: Improve fitting convergence behavior for...	Closed	07/28/2014	
Related to GammaLib - Action # 1304: Improve fitting convergence behavior for...	Closed	07/28/2014	

## History

### #1 - 07/25/2014 10:18 PM - Knödseder Jürgen

- Description updated

### #2 - 07/25/2014 10:24 PM - Knödseder Jürgen

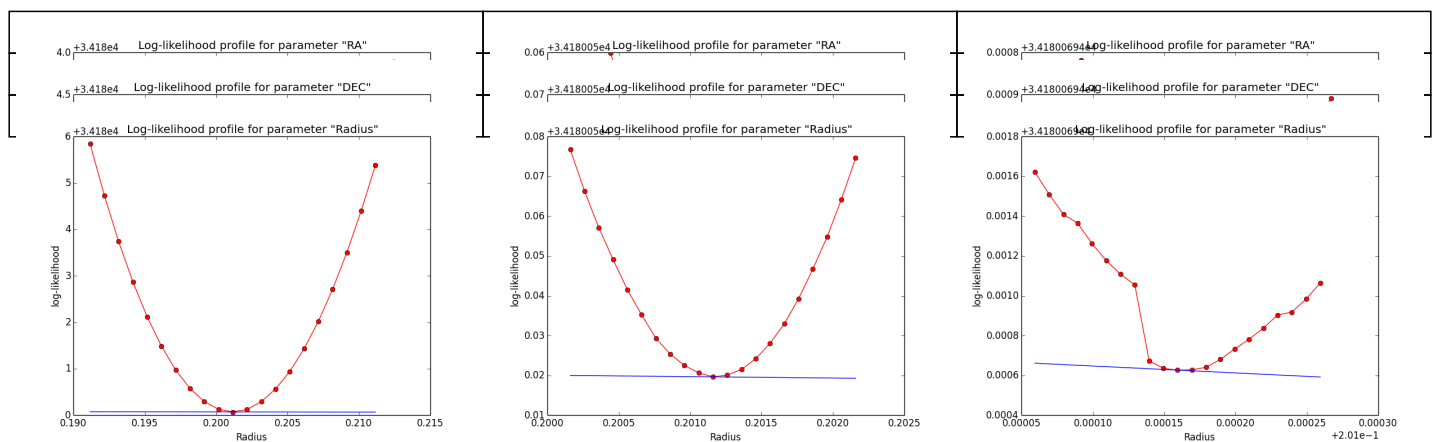
- Description updated

### #3 - 07/25/2014 10:30 PM - Knödseder Jürgen

- File disk\_unbin\_scale3\_DEC.png added
- File disk\_unbin\_scale3\_RA.png added
- File disk\_unbin\_scale3\_Radius.png added
- File disk\_unbin\_scale4\_DEC.png added
- File disk\_unbin\_scale4\_RA.png added
- File disk\_unbin\_scale4\_Radius.png added
- File disk\_unbin\_scale5\_DEC.png added
- File disk\_unbin\_scale5\_RA.png added
- File disk\_unbin\_scale5\_Radius.png added

## Problem description

Here a number of log-likelihood profiles for the disk parameters RA (top), DEC (middle), and Radius (bottom). The plots from the left to the right show successive zooms with step size  $1e-3$  (left),  $1e-4$  (centre) and  $1e-5$  (right). The red line connects the computed log-likelihood values (red dots), the blue line indicates the gradient obtained by the optimizer. For gradient computation, a step size of  $1e-5$  has been used. Obviously, the log-likelihood profile is "nice" at a scale of  $1e-3$ , becomes distorted at a scale of  $1e-4$  and becomes very noisy at a scale of  $1e-5$ .



### #4 - 07/25/2014 10:37 PM - Knödseder Jürgen

- Status changed from New to In Progress
- Assigned To set to Knödseder Jürgen

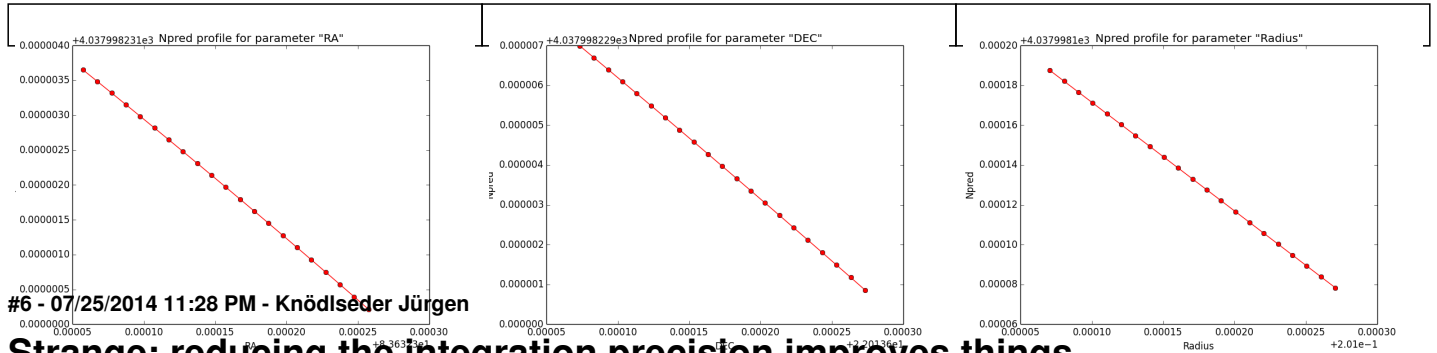
Reducing the numerical differentiation step-size did not really change anything. So although the profile looks smooth on a large scale, it is not precise enough for accurate numerical derivatives that let the algorithm converge properly.

### #5 - 07/25/2014 10:49 PM - Knödseder Jürgen

- File disk\_unbin\_Npred\_DEC.png added
- File disk\_unbin\_Npred\_RA.png added
- File disk\_unbin\_Npred\_Radius.png added
- % Done changed from 0 to 10

# Npred computation not the problem

I checked that the Npred term is not the problem in the unbinned analysis. Below are Npred profiles for a step size of 1e-5 in all three parameters. Everything is smooth. This implies that it must be the probability density computation for the individual events that creates the structure in the log-likelihood profiles.



#6 - 07/25/2014 11:28 PM - Knödseder Jürgen

## Strange: reducing the integration precision improves things

The event probability computation is done in a system centred on the radial model that is integrated radially and azimuthally (in a spherical system). The integration precisions in both dimensions has been set to 1e-5. I reduced the precision to 1e-2, which gave the following result:

```
>Iteration 0: -logL=34178.844, Lambda=1.0e-03
>Iteration 1: -logL=34176.559, Lambda=1.0e-03, delta=2.285, max(|grad|)=20.694989 [DEC:1]
>Iteration 2: -logL=34176.548, Lambda=1.0e-04, delta=0.011, max(|grad|)=6.348015 [RA:0]
>Iteration 3: -logL=34176.547, Lambda=1.0e-05, delta=0.000, max(|grad|)=-1.458495 [RA:0]
>Iteration 4: -logL=34176.547, Lambda=1.0e-06, delta=0.000, max(|grad|)=0.392503 [RA:0]
>Iteration 5: -logL=34176.547, Lambda=1.0e-07, delta=0.000, max(|grad|)=-0.073415 [RA:0]
Iteration 6: -logL=34176.547, Lambda=1.0e-08, delta=-0.000, max(|grad|)=0.016422 [RA:0] (stalled)
Iteration 7: -logL=34176.547, Lambda=1.0e-07, delta=-0.000, max(|grad|)=-0.003630 [RA:0] (stalled)
Iteration 8: -logL=34176.547, Lambda=1.0e-06, delta=-0.000, max(|grad|)=-0.002001 [Radius:2] (stalled)
Iteration 9: -logL=34176.547, Lambda=1.0e-05, delta=-0.000, max(|grad|)=0.001361 [Radius:2] (stalled)
Iteration 10: -logL=34176.547, Lambda=1.0e-04, delta=-0.000, max(|grad|)=0.001189 [Radius:2] (stalled)
>Iteration 11: -logL=34176.547, Lambda=1.0e-03, delta=0.000, max(|grad|)=0.000281 [DEC:1]
=== GObservations ===
Number of observations .....: 1
Number of predicted events : 4038
=== GModelSky ===
Number of spatial par's ....: 3
RA .....: 83.6325 +/- 0.00393904 [-360,360] deg (free,scale=1)
DEC .....: 22.0138 +/- 0.00327544 [-90,90] deg (free,scale=1)
Radius .....: 0.201183 +/- 0.00305945 [0.01,10] deg (free,scale=1)
Number of spectral par's ...: 3
Prefactor .....: 5.45629e-16 +/- 1.99354e-17 [1e-23,1e-13] ph/cm2/s/MeV (free,scale=1e-16,gradient)
Index .....: -2.45898 +/- 0.0267017 [-0,-5] (free,scale=-1,gradient)
=== GCTAModelRadialAcceptance ===
Sigma .....: 3.05836 +/- 0.0397935 [0.01,10] deg2 (free,scale=1,gradient)
Number of spectral par's ...: 3
Prefactor .....: 5.99534e-05 +/- 1.76003e-06 [0,0.001] ph/cm2/s/MeV (free,scale=1e-06,gradient)
Index .....: -1.85299 +/- 0.0164762 [-0,-5] (free,scale=-1,gradient)
```

This takes less iterations, and the gradients are smaller than initially.

#7 - 07/25/2014 11:29 PM - Knödlseider Jürgen

- Description updated

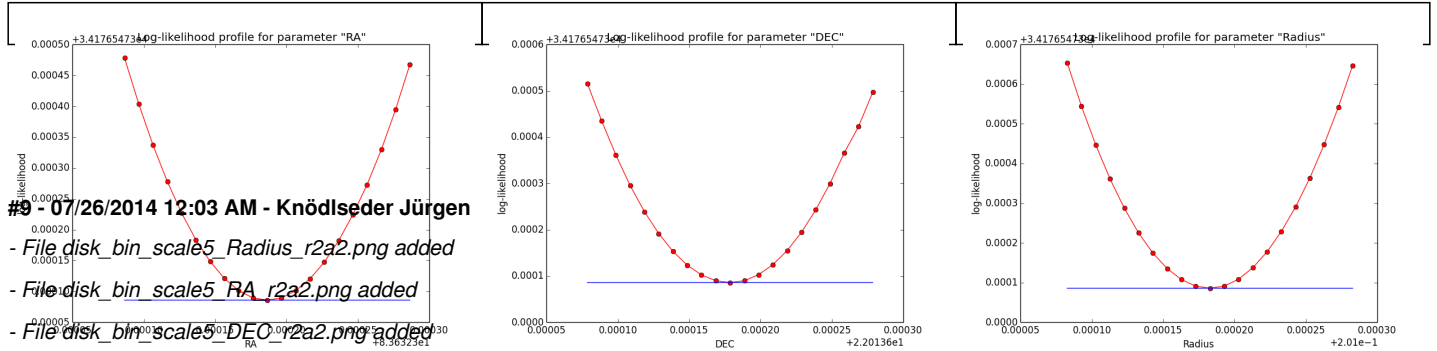
#8 - 07/25/2014 11:32 PM - Knödlseider Jürgen

- File disk\_unbin\_scale5\_DE\_r2a2.png added

- File disk\_unbin\_scale5\_RA\_r2a2.png added

- File disk\_unbin\_scale5\_Radius\_r2a2.png added

Below the log-likelihood profiles with a step size of  $1e-5$  for the reduced relative integration precision of  $1e-2$ . This seems counter intuitive, but with less precision the profiles are smoother!



#9 - 07/26/2014 12:03 AM - Knödlseider Jürgen

- File disk\_bin\_scale5\_Radius\_r2a2.png added

- File disk\_bin\_scale5\_RA\_r2a2.png added

- File disk\_bin\_scale5\_DEC\_r2a2.png added

## Binned also better for $1e-2$

Here now a comparison of the binned analysis, first with the old precision of  $1e-5$ , and then with the new precision of  $1e-2$ . The fit obviously converges much faster and smoother with the reduced integration precision of  $1e-2$ . The results are pretty much the same.

### Integration precision $1e-5$

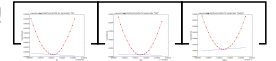
```
>Iteration 0: -logL=18456.210, Lambda=1.0e-03
>Iteration 1: -logL=18452.575, Lambda=1.0e-03, delta=3.635, max(|grad|)=-28.343815 [RA:0]
>Iteration 2: -logL=18452.529, Lambda=1.0e-04, delta=0.045, max(|grad|)=-16.720985 [DEC:1]
>Iteration 3: -logL=18452.529, Lambda=1.0e-05, delta=0.000, max(|grad|)=6.820850 [DEC:1]
>Iteration 4: -logL=18452.528, Lambda=1.0e-06, delta=0.001, max(|grad|)=-2.600544 [RA:0]
Iteration 5: -logL=18452.528, Lambda=1.0e-07, delta=-0.000, max(|grad|)=3.205072 [RA:0] (stalled)
Iteration 6: -logL=18452.528, Lambda=1.0e-06, delta=-0.000, max(|grad|)=3.205211 [RA:0] (stalled)
Iteration 7: -logL=18452.528, Lambda=1.0e-05, delta=-0.000, max(|grad|)=3.205105 [RA:0] (stalled)
Iteration 8: -logL=18452.528, Lambda=1.0e-04, delta=-0.000, max(|grad|)=3.204754 [RA:0] (stalled)
Iteration 9: -logL=18452.528, Lambda=1.0e-03, delta=-0.000, max(|grad|)=3.248355 [RA:0] (stalled)
Iteration 10: -logL=18452.528, Lambda=1.0e-02, delta=-0.000, max(|grad|)=3.240016 [RA:0] (stalled)
Iteration 11: -logL=18452.528, Lambda=1.0e-01, delta=-0.000, max(|grad|)=3.080144 [RA:0] (stalled)
>Iteration 12: -logL=18452.528, Lambda=1.0e+00, delta=0.000, max(|grad|)=-1.595976 [Radius:2]
>Iteration 13: -logL=18452.528, Lambda=1.0e-01, delta=0.000, max(|grad|)=2.660137 [DEC:1]
Iteration 14: -logL=18452.528, Lambda=1.0e-02, delta=-0.000, max(|grad|)=0.518520 [DEC:1] (stalled)
Iteration 15: -logL=18452.528, Lambda=1.0e-01, delta=-0.000, max(|grad|)=2.844791 [RA:0] (stalled)
>Iteration 16: -logL=18452.528, Lambda=1.0e+00, delta=0.000, max(|grad|)=2.905791 [RA:0]
Iteration 17: -logL=18452.528, Lambda=1.0e-01, delta=-0.000, max(|grad|)=-2.042002 [RA:0] (stalled)
Iteration 18: -logL=18452.528, Lambda=1.0e+00, delta=-0.000, max(|grad|)=-1.560911 [RA:0] (stalled)
>Iteration 19: -logL=18452.528, Lambda=1.0e+01, delta=0.000, max(|grad|)=-2.206019 [DEC:1]
=== GObservations ===
Number of observations ....: 1
Number of predicted events : 3687
=== GModelSky ===
Number of spatial par's ....: 3
RA .....: 83.6324 +/- 0.00390577 [-360,360] deg (free,scale=1)
DEC .....: 22.0143 +/- 0.00329028 [-90,90] deg (free,scale=1)
Radius .....: 0.201591 +/- 0.00304322 [0.01,10] deg (free,scale=1)
Number of spectral par's ...: 3
Prefactor .....: 5.41603e-16 +/- 1.98302e-17 [1e-23,1e-13] ph/cm2/s/MeV (free,scale=1e-16,gradient)
Index .....: -2.45653 +/- 0.0267726 [-0,-5] (free,scale=-1,gradient)
=== GCTAModelRadialAcceptance ===
Number of radial par's .....: 1
Sigma .....: 3.15909 +/- 0.0857976 [0.01,10] deg2 (free,scale=1,gradient)
Number of spectral par's ...: 3
Prefactor .....: 5.88695e-05 +/- 1.90709e-06 [0,0.001] ph/cm2/s/MeV (free,scale=1e-06,gradient)
```

Index .....: -1.85371 +/- 0.0175121 [-0,-5] (free,scale=-1,gradient)

## Integration precision 1e-2

```
>Iteration 0: -logL=18456.397, Lambda=1.0e-03
>Iteration 1: -logL=18452.583, Lambda=1.0e-03, delta=3.814, max(|grad|)=-20.005022 [RA:0]
>Iteration 2: -logL=18452.538, Lambda=1.0e-04, delta=0.046, max(|grad|)=5.343857 [RA:0]
>Iteration 3: -logL=18452.537, Lambda=1.0e-05, delta=0.000, max(|grad|)=-1.070713 [RA:0]
>Iteration 4: -logL=18452.537, Lambda=1.0e-06, delta=0.000, max(|grad|)=0.358104 [RA:0]
>Iteration 5: -logL=18452.537, Lambda=1.0e-07, delta=0.000, max(|grad|)=0.263873 [Radius:2]
=== GObservations ===
Number of observations .....: 1
Number of predicted events : 3687
=== GModelSky ===
Number of spatial par's ....: 3
RA .....: 83.6325 +/- 0.00390558 [-360,360] deg (free,scale=1)
DEC .....: 22.0143 +/- 0.00329027 [-90,90] deg (free,scale=1)
Radius .....: 0.201619 +/- 0.00304471 [0.01,10] deg (free,scale=1)
Number of spectral par's ...: 3
Prefactor .....: 5.39378e-16 +/- 1.97492e-17 [1e-23,1e-13] ph/cm2/s/MeV (free,scale=1e-16,gradient)
Index .....: -2.45609 +/- 0.0267721 [-0,-5] (free,scale=-1,gradient)
=== GCTAModelRadialAcceptance ===
Number of radial par's .....: 1
Sigma .....: 3.15907 +/- 0.0857955 [0.01,10] deg2 (free,scale=1,gradient)
Number of spectral par's ...: 3
Prefactor .....: 5.88702e-05 +/- 1.9071e-06 [0,0.001] ph/cm2/s/MeV (free,scale=1e-06,gradient)
Index .....: -1.85371 +/- 0.017512 [-0,-5] (free,scale=-1,gradient)
```

And here the log-likelihood profiles for binned analysis with an integration precision of 1e-2 at a scale of 1e-5: very smooth!



#10 - 07/26/2014 12:18 AM - Knödlseeder Jürgen

- File gauss\_unbin\_scale5\_RA\_r2a2.png added

- File gauss\_unbin\_scale5\_RA\_r5a5.png added

- % Done changed from 10 to 20

# Gaussian model has problems at all scales!!!

Crosschecking then the unbinned Gaussian model as another case I recognized that also here the log-likelihood profiles are bad, and even for an integration precision of 1e-2 have a kink. Here the iterations of the 1e-5 and 1e-2 precision runs. Fitting results were the same for both runs.

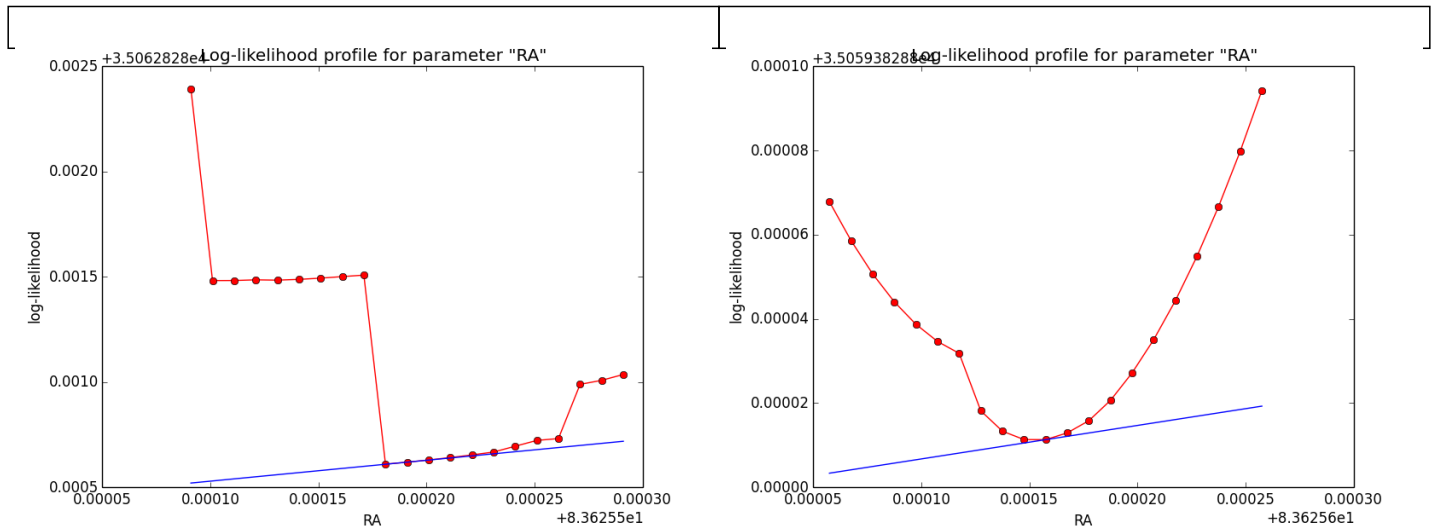
## Integration precision 1e-5

```
>Iteration 0: -logL=35065.905, Lambda=1.0e-03
>Iteration 1: -logL=35062.850, Lambda=1.0e-03, delta=3.055, max(|grad|)=26.700169 [Sigma:2]
>Iteration 2: -logL=35062.829, Lambda=1.0e-04, delta=0.022, max(|grad|)=44.862488 [DEC:1]
Iteration 3: -logL=35062.829, Lambda=1.0e-05, delta=-0.126, max(|grad|)=-39.775759 [Sigma:2] (stalled)
Iteration 4: -logL=35062.829, Lambda=1.0e-04, delta=-0.126, max(|grad|)=-39.772432 [Sigma:2] (stalled)
Iteration 5: -logL=35062.829, Lambda=1.0e-03, delta=-0.126, max(|grad|)=-39.687673 [Sigma:2] (stalled)
Iteration 6: -logL=35062.829, Lambda=1.0e-02, delta=-0.124, max(|grad|)=-39.369992 [Sigma:2] (stalled)
Iteration 7: -logL=35062.829, Lambda=1.0e-01, delta=-0.104, max(|grad|)=-71.231283 [Sigma:2] (stalled)
Iteration 8: -logL=35062.829, Lambda=1.0e+00, delta=-0.039, max(|grad|)=-21.992135 [Sigma:2] (stalled)
Iteration 9: -logL=35062.829, Lambda=1.0e+01, delta=-0.002, max(|grad|)=-6.135290 [Sigma:2] (stalled)
Iteration 10: -logL=35062.829, Lambda=1.0e+02, delta=-0.000, max(|grad|)=2.564888 [RA:0] (stalled)
Iteration 11: -logL=35062.829, Lambda=1.0e+03, delta=-0.000, max(|grad|)=43.240077 [Sigma:2] (stalled)
Iteration 12: -logL=35062.829, Lambda=1.0e+04, delta=-0.000, max(|grad|)=44.858612 [DEC:1] (stalled)
```

## Integration precision 1e-2

```
>Iteration 0: -logL=35062.491, Lambda=1.0e-03
>Iteration 1: -logL=35059.403, Lambda=1.0e-03, delta=3.087, max(|grad|)=27.067091 [Sigma:2]
>Iteration 2: -logL=35059.383, Lambda=1.0e-04, delta=0.020, max(|grad|)=-2.179918 [Sigma:2]
>Iteration 3: -logL=35059.383, Lambda=1.0e-05, delta=0.000, max(|grad|)=-0.884052 [RA:0]
Iteration 4: -logL=35059.383, Lambda=1.0e-06, delta=-0.000, max(|grad|)=0.904869 [RA:0] (stalled)
Iteration 5: -logL=35059.383, Lambda=1.0e-05, delta=-0.000, max(|grad|)=0.904726 [RA:0] (stalled)
Iteration 6: -logL=35059.383, Lambda=1.0e-04, delta=-0.000, max(|grad|)=0.904932 [RA:0] (stalled)
Iteration 7: -logL=35059.383, Lambda=1.0e-03, delta=-0.000, max(|grad|)=0.904089 [RA:0] (stalled)
Iteration 8: -logL=35059.383, Lambda=1.0e-02, delta=-0.000, max(|grad|)=0.893860 [RA:0] (stalled)
Iteration 9: -logL=35059.383, Lambda=1.0e-01, delta=-0.000, max(|grad|)=0.803572 [RA:0] (stalled)
>Iteration 10: -logL=35059.383, Lambda=1.0e+00, delta=0.000, max(|grad|)=0.253246 [RA:0]
>Iteration 11: -logL=35059.383, Lambda=1.0e-01, delta=0.000, max(|grad|)=-0.104525 [RA:0]
>Iteration 12: -logL=35059.383, Lambda=1.0e-02, delta=0.000, max(|grad|)=0.079574 [RA:0]
```

And here the log-likelihood profiles, on the left for a precision of 1e-5, on the right for a precision of 1e-2.



## Changing the integration scheme does not improve things

I implemented an alternative radial model integration method that uses a coordinate system centred on the Psf instead of the former system that was centred on the model. The integration region is the same, yet once the integration has as symmetry point the Psf centre while in the other case it has the model centre as symmetry point. This may lead to numerical differences.

To check the precision between both computation methods, I computed for the **disk model** the lrf value for each event with both methods and dumped any difference larger than 1% into the console (I've chosen 1% as otherwise I got a quite verbose run). Here the outcome (the fit was done using the model-based integration; the step-size for gradient computation was  $2e-4$ , the precision for integration was set to  $1e-6$  for both methods):

```
Psf-based=8.41043e+12; Model-based=8.31804e+12; Psf-Model=9.23986e+10; Psf/Model=1.01111
Psf-based=1.79475e+14; Model-based=1.77137e+14; Psf-Model=2.33874e+12; Psf/Model=1.0132
Psf-based=9.60219e+12; Model-based=9.47734e+12; Psf-Model=1.24851e+11; Psf/Model=1.01317
>Iteration 0: -logL=34182.387, Lambda=1.0e-03
Psf-based=9.51493e+12; Model-based=9.39039e+12; Psf-Model=1.24537e+11; Psf/Model=1.01326
>Iteration 1: -logL=34180.097, Lambda=1.0e-03, delta=2.290, max(|grad|)=23.256112 [DEC:1]
>Iteration 2: -logL=34180.086, Lambda=1.0e-04, delta=0.011, max(|grad|)=6.080755 [RA:0]
>Iteration 3: -logL=34180.086, Lambda=1.0e-05, delta=0.000, max(|grad|)=1.558499 [Radius:2]
>Iteration 4: -logL=34180.086, Lambda=1.0e-06, delta=0.000, max(|grad|)=-0.566938 [Radius:2]
Iteration 5: -logL=34180.086, Lambda=1.0e-07, delta=-0.000, max(|grad|)=0.183964 [Radius:2] (stalled)
Iteration 6: -logL=34180.086, Lambda=1.0e-06, delta=-0.000, max(|grad|)=0.183985 [Radius:2] (stalled)
Iteration 7: -logL=34180.086, Lambda=1.0e-05, delta=-0.000, max(|grad|)=0.183965 [Radius:2] (stalled)
Iteration 8: -logL=34180.086, Lambda=1.0e-04, delta=-0.000, max(|grad|)=0.183878 [Radius:2] (stalled)
Iteration 9: -logL=34180.086, Lambda=1.0e-03, delta=-0.000, max(|grad|)=0.183239 [Radius:2] (stalled)
Iteration 10: -logL=34180.086, Lambda=1.0e-02, delta=-0.000, max(|grad|)=0.176919 [Radius:2] (stalled)
Iteration 11: -logL=34180.086, Lambda=1.0e-01, delta=-0.000, max(|grad|)=0.119540 [Radius:2] (stalled)
Iteration 12: -logL=34180.086, Lambda=1.0e+00, delta=-0.000, max(|grad|)=0.153442 [RA:0] (stalled)
Iteration 13: -logL=34180.086, Lambda=1.0e+01, delta=-0.000, max(|grad|)=-0.149491 [Radius:2] (stalled)
Iteration 14: -logL=34180.086, Lambda=1.0e+02, delta=-0.000, max(|grad|)=-0.147830 [Radius:2] (stalled)
=== GOptimizerLM ===
Optimized function value ...: 34180.086
```

And here the console dump in case that the Psf-based integration was used (the difference with respect to the former is that now the lrf was used from the Psf-based integration, leading to a somewhat different iteration scheme):

```
Psf-based=8.41043e+12; Model-based=8.31804e+12; Psf-Model=9.23986e+10; Psf/Model=1.01111
Psf-based=1.79475e+14; Model-based=1.77137e+14; Psf-Model=2.33874e+12; Psf/Model=1.0132
Psf-based=9.60219e+12; Model-based=9.47734e+12; Psf-Model=1.24851e+11; Psf/Model=1.01317
>Iteration 0: -logL=34182.431, Lambda=1.0e-03
Psf-based=1.77896e+14; Model-based=1.75561e+14; Psf-Model=2.33491e+12; Psf/Model=1.0133
Psf-based=2.90648e+13; Model-based=2.86749e+13; Psf-Model=3.89811e+11; Psf/Model=1.01359
>Iteration 1: -logL=34180.131, Lambda=1.0e-03, delta=2.300, max(|grad|)=58.238133 [RA:0]
Iteration 2: -logL=34180.131, Lambda=1.0e-04, delta=-0.017, max(|grad|)=-64.345341 [RA:0] (stalled)
Iteration 3: -logL=34180.131, Lambda=1.0e-03, delta=-0.017, max(|grad|)=-63.541598 [RA:0] (stalled)
Iteration 4: -logL=34180.131, Lambda=1.0e-02, delta=-0.016, max(|grad|)=-68.675928 [RA:0] (stalled)
Iteration 5: -logL=34180.131, Lambda=1.0e-01, delta=-0.016, max(|grad|)=-51.220461 [RA:0] (stalled)
>Iteration 6: -logL=34180.123, Lambda=1.0e+00, delta=0.008, max(|grad|)=-22.486019 [DEC:1]
Psf-based=9.52107e+12; Model-based=9.39527e+12; Psf-Model=1.25801e+11; Psf/Model=1.01339
>Iteration 7: -logL=34180.110, Lambda=1.0e-01, delta=0.013, max(|grad|)=39.522341 [Radius:2]
Psf-based=5.02533e+13; Model-based=4.95867e+13; Psf-Model=6.66573e+11; Psf/Model=1.01344
Iteration 8: -logL=34180.110, Lambda=1.0e-01, delta=-0.025, max(|grad|)=-42.493911 [Radius:2] (stalled)
Iteration 9: -logL=34180.110, Lambda=1.0e-02, delta=-0.023, max(|grad|)=-55.226406 [Radius:2] (stalled)
Psf-based=5.0183e+13; Model-based=4.95e+13; Psf-Model=6.82977e+11; Psf/Model=1.0138
Psf-based=5.01623e+13; Model-based=4.94955e+13; Psf-Model=6.66865e+11; Psf/Model=1.01347
>Iteration 10: -logL=34180.107, Lambda=1.0e+00, delta=0.003, max(|grad|)=-54.111179 [DEC:1]
Psf-based=1.77426e+14; Model-based=1.75139e+14; Psf-Model=2.28669e+12; Psf/Model=1.01306
Psf-based=1.86115e+13; Model-based=1.83495e+13; Psf-Model=2.6203e+11; Psf/Model=1.01428
Iteration 11: -logL=34180.107, Lambda=1.0e-01, delta=-0.031, max(|grad|)=45.835401 [DEC:1] (stalled)
Iteration 12: -logL=34180.107, Lambda=1.0e+00, delta=-0.017, max(|grad|)=62.316907 [DEC:1] (stalled)
Psf-based=5.01642e+13; Model-based=4.94901e+13; Psf-Model=6.74144e+11; Psf/Model=1.01362
Iteration 13: -logL=34180.107, Lambda=1.0e+01, delta=-0.020, max(|grad|)=33.570227 [DEC:1] (stalled)
Psf-based=5.01786e+13; Model-based=4.94984e+13; Psf-Model=6.80154e+11; Psf/Model=1.01374
Psf-based=5.0158e+13; Model-based=4.94939e+13; Psf-Model=6.64141e+11; Psf/Model=1.01342
Iteration 14: -logL=34180.107, Lambda=1.0e+02, delta=-0.004, max(|grad|)=-52.946653 [DEC:1] (stalled)
Psf-based=5.01825e+13; Model-based=4.94998e+13; Psf-Model=6.8269e+11; Psf/Model=1.01379
Psf-based=5.01619e+13; Model-based=4.94953e+13; Psf-Model=6.66588e+11; Psf/Model=1.01347
>Iteration 15: -logL=34180.107, Lambda=1.0e+03, delta=0.000, max(|grad|)=-54.562612 [DEC:1]
Psf-based=5.01787e+13; Model-based=4.94984e+13; Psf-Model=6.80328e+11; Psf/Model=1.01374
Psf-based=5.01582e+13; Model-based=4.94938e+13; Psf-Model=6.64307e+11; Psf/Model=1.01342
...
```



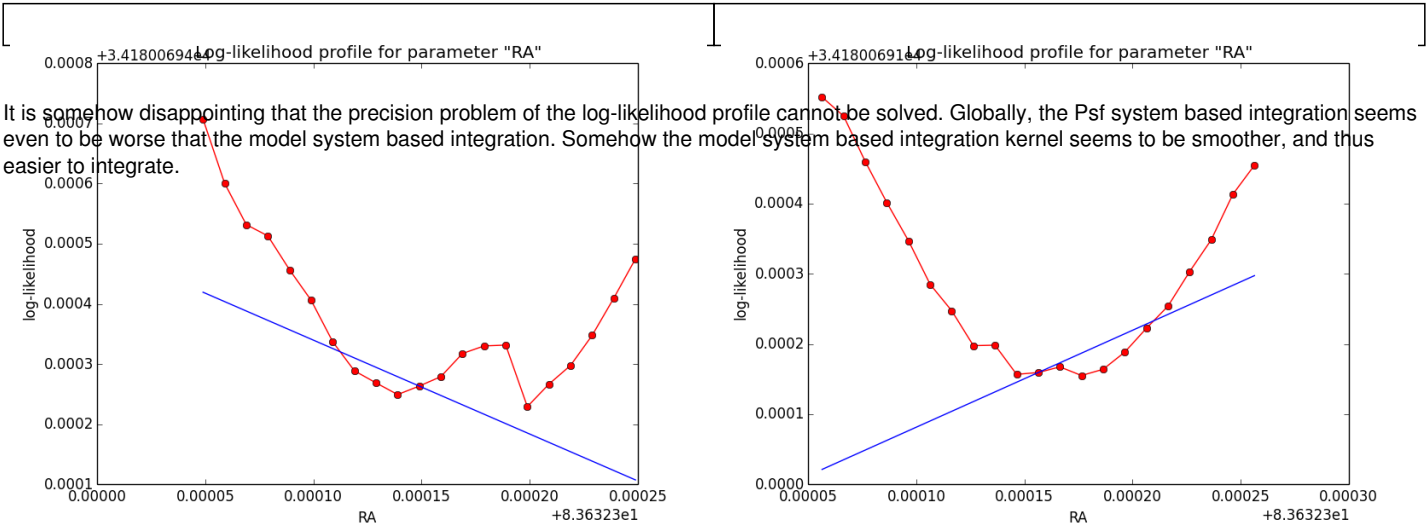


#14 - 07/27/2014 10:56 PM - Knödlseider Jürgen

- File disk\_unbin-model\_scale5\_RA\_prec5\_grad0002.png added
- File disk\_unbin-model-switch\_scale5\_RA\_prec5\_grad0002.png added
- % Done changed from 20 to 40

## Splitting the integral into two parts

I now implemented a split of the integral into two parts, one to the left and another to the right of the kink (which is fully predictable). Using the model system based integration, an integration precision of  $1e-5$  and a gradient step size of  $h=2e-4$ , I obtained the following log-likelihood profiles (for a step size of  $1e-5$ ). Left is before split into two part, right is afterwards. Including the split results in a smoother, though not perfect profile.



#15 - 07/27/2014 11:03 PM - Knödlseider Jürgen


## Testing a more robust derivative scheme

In case that the log-likelihood profile cannot be made smoother, a more robust numerical derivation scheme is needed. I found this very interesting page [web page](#) from Pavel Holoborodko which provides smooth noise-robust differentiators. They require more function evaluations, but overall this may still lead to a gain in computation speed if less iterations would be needed. I played around with a couple of the different schemes, characterized by a degree and a filter length. Below some results for an unbinned analysis. N/A in the degree and filter length column refer to the original difference method.

Model	Degree	Filter length	Precision	Step size	Iterations	Max. gradient
Disk	N/A	N/A	1e-5	2e-4	15	1.380502 [RA:0]
	2	5	1e-5	2e-4	16	0.405655 [RA:0]
	2	7	1e-5	2e-4	8	0.000851 [RA:0]
	4	7	1e-5	2e-4	13	0.606999 [RA:0]
	2	5	1e-5	1e-5	12	-4.197968 [Radius:2]
	N/A	N/A	1e-5	1e-3	14	-0.115223 [Radius:2]
	2	5	1e-5	1e-3	6	-0.062993 [RA:0]
	2	7	1e-5	1e-3	17	0.122399 [RA:0]
	2	5	1e-2	1e-3	7	-0.004124 [RA:0]
	N/A	N/A	1e-2	1e-3	13	2.470741 [Radius:2]
Gauss	N/A	N/A	1e-5	1e-3		
	2	5	1e-5	1e-3	10	-0.501377 [RA:0]
	2	7	1e-5	1e-3	18	0.166057 [DEC:1]
	2	9	1e-5	1e-3	18	-0.169905 [RA:0]
	2	5	1e-2	1e-3	4	-0.024501 [Sigma:2]

	N/A	N/A	1e-2	1e-3	21	64.872221 [Sigma:2]
Shell	N/A	N/A	1e-2	1e-3		
	2	5	1e-2	1e-3	32	25445.758402 [Radius:2]
	2	5	1e-5	1e-3	26	25723.513680 [Radius:2]

There is a trend of smaller maximum gradients with the noise-robust derivatives, in particular for larger filter lengths. However, the only thing that really helps - at least for the disk and the Gaussian model - is setting the integration precision to 1e-2, which now gives very smooth log-likelihood profiles (the kinks shown above for the Gaussian model disappeared after introducing the split integration intervals). Then, the noise-robust estimator with a degree of 2 and a filter length of 5 does a good job.

For the shell, however, nothing seems really to work  The maximum likelihood is probably not reached.

#16 - 07/28/2014 08:12 AM - Knödlseeder Jürgen

- File gauss\_5iter.png added
- File gauss\_6iter.png added
- File gauss\_7iter.png added
- File disk\_7iter\_prec5-diff.png added
- File disk\_6iter\_prec5-diff.png added
- File disk\_5iter\_prec5-diff.png added

## Issue solved?

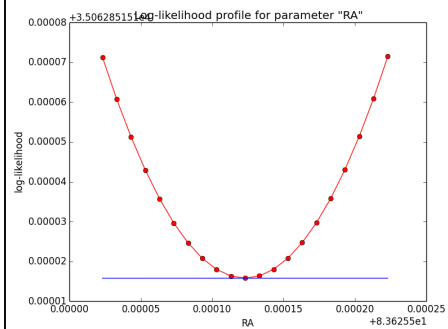
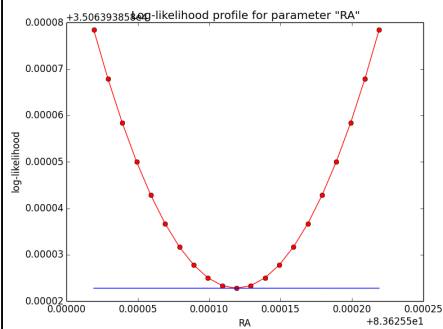
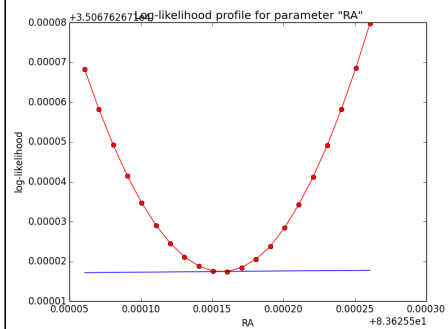
Thinking again over it, I'm wondering whether the "noise" may not come from the fact that the integration precision is not guaranteed. All numerical integration methods do some iterative value refinement, and stop when the difference between subsequent iterations falls below a limit. This may lead to different numbers of iterations depending on details of the setup. When now a parameter is varied, the number of iterations may change, leading to a different value at the end.

I checked this idea using the gauss model, unbinned analysis and model system based integration. For a precision of 1e-2, all rho integrations needed 5 iterations, while for a precision of 1e-5 the number of iterations varied between 5 and 7. The calculations were done using the (2,5) noise-robust derivatives for a step-size h=1e-3.

Integration iterations	Fit iterations	logL	Max. gradient
5	4	35067.627	-0.028632 [Sigma:2]
6	6	35063.939	-0.000331 [Sigma:2]
7	6	35062.852	-0.000313 [Sigma:2]

Here the corresponding log-likelihood profiles (from 5 iterations on the left to 7 iterations on the right):



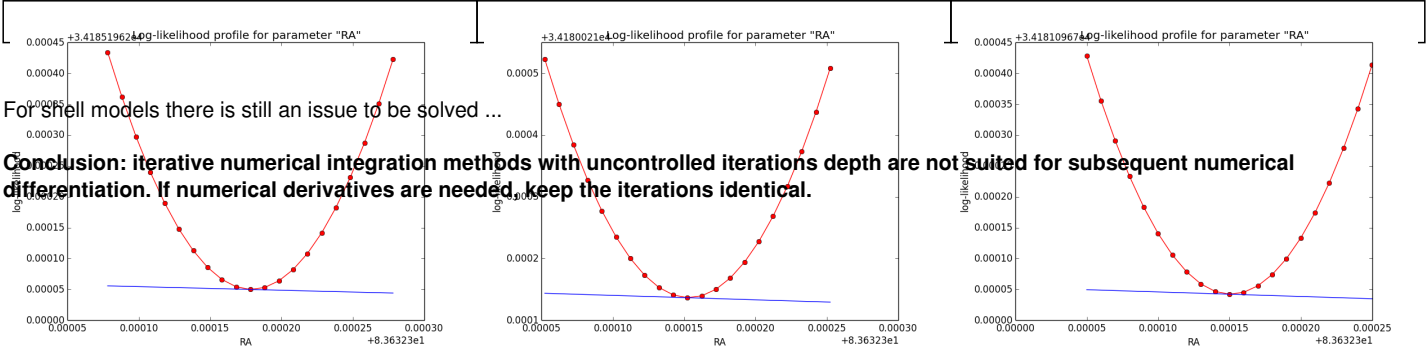


Now the same using the simple difference derivatives for a step-size of  $h=1e-5$ .

Integration iterations	Fit iterations	logL	Max. gradient
5	5	35067.627	0.003536 [Sigma:2]
6	4	35063.939	-0.026879 [Sigma:2]
7	4	35062.852	-0.025805 [Sigma:2]

This looks pretty good. Good a quick convergence even for  $h=1e-5$ . Now the same (simple difference derivatives, step-size of  $h=1e-5$ ) for the disk model:

Integration iterations	Fit iterations	logL	Max. gradient
5	5	34185.196	-0.058426 [RA:0]
6	5	34181.097	-0.073399 [RA:0]
7	5	34180.021	-0.071788 [RA:0]



#17 - 07/28/2014 10:02 AM - Knödlseeder Jürgen  
 - File disk\_psf-switch\_5iter\_prec5-diff.png added

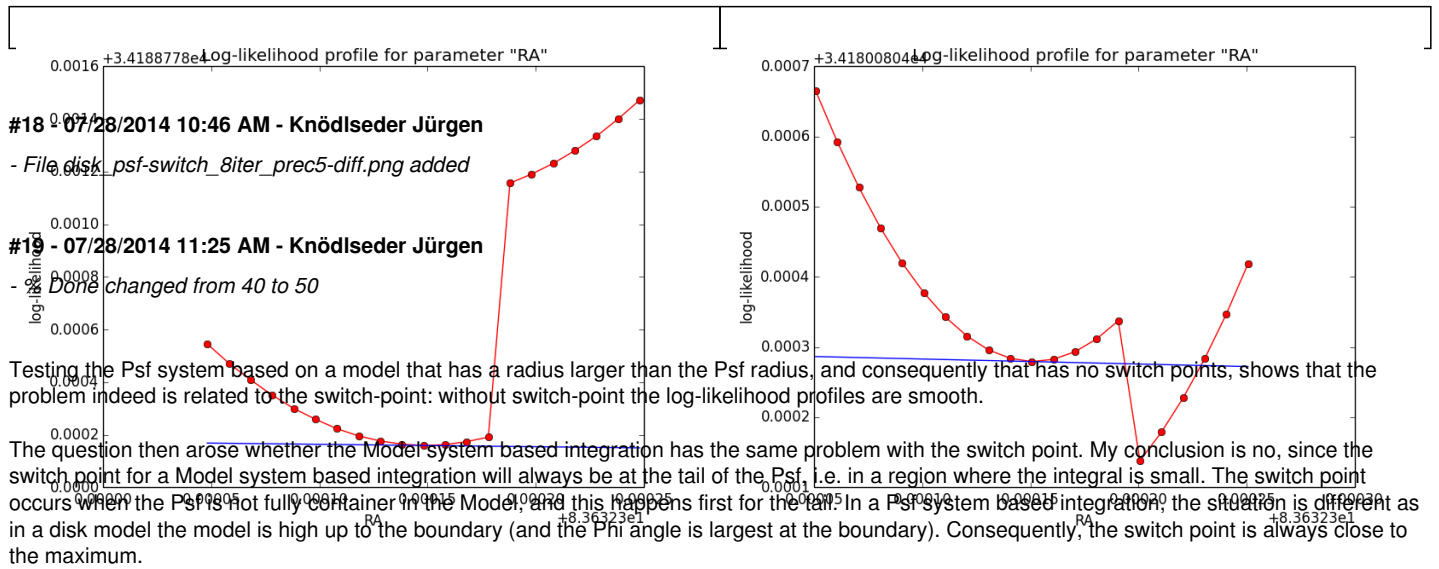
## Still some trouble with Psf system based integration

I now checked the same philosophy for the Psf system based integration. I started with the disk model.

It turns out that with Psf system based integration there is still a problem: the log-likelihood profile is now smooth, but there appears a clear step near the minimum in the profile (see left plot below for 5 iterations). This step changes (see right plot for 8 iterations) with an increasing number of iterations for the integration. This could indicate that the step is related to the limited precision of the integration.

As with the change of the parameters the switch point between full and partial containment of the model within the Psf is changing, the function evaluation in the left and right part of the switch point changes, being a source for discontinuities (as every side will have it's own intrinsic precision).

Integration iterations	Fit iterations	logL	Max. gradient
5	20	34188.778	0.493484 [Radius:2]
6	18	34181.029	36.308235 [Radius:2]
8	5	34180.081	0.073235 [Radius:2]



**Conclusion: a Psf system based integration has poorer properties due to the occurrence of the switch point at function maximum.**

I now switch-off the Psf system based integration for unbinned and binned analysis.

**#20 - 07/28/2014 05:52 PM - Knödlseider Jürgen**

- Target version set to 00-09-00

- % Done changed from 50 to 80

The shell model still remains a tricky model to integrate, and so far the code is not yet fully satisfactory. Some more investigations are needed.

For the moment let's stay with the code as is. Here now a check of the Irf computation for all model types and analysis. I show here the computing time needed to estimate the total number of counts in a model, and in parentheses the relative precision achieved. Results are shown for unbinned, binned and stacked analysis. For unbinned, the Npred methods are tested, for binned, the Irf methods are tested. Hence this provides a complete measure of the computing speed (in seconds) and of the precision ((estimated-expected)/expected) of the CTA response computation. The test is done using the script benchmark\_irf\_computation.py.

Model	Unbinned	Binned	Stacked
Point	0.0 (0.3%)	3.8 (0.8%)	3.0 (0.8%)
Disk	0.1 (0.4%)	5.4 (1.3%)	4.0 (1.1%)
Gauss	0.1 (0.2%)	18.4 (1.1%)	10.2 (0.9%)
Shell	0.1 (0.3%)	7.0 (0.7%)	5.1 (1.1%)
Ellipse	0.1 (0.1%)	35.8 (1.0%)	30.3 (0.6%)
Diffuse	0.2 (-0.5%)	228.8 (0.0%)	96.0 (0.5%)

## #21 - 07/28/2014 05:57 PM - Knödseder Jürgen

And now a benchmark on all model fits, done using `benchmark_ml_fitting.py`. This checks whether the gradient computations are okay, and whether the fitting algorithm converges reasonably. Bad convergence is indicated by an excessive number of iterations, and these results are marked in bold in the summary table below. Shown are the elapsed time, the number of iterations needed, the log-likelihood, and the fitted source parameters (given in the order they appear in the model).

Obviously, the shell model needs improvements, as well as the ellipse models. I created issue #1303 to follow-up on the shell model improvement and issue #1304 to follow-up on the ellipse model improvement. Also the stacked analysis for the disk is not yet optimal. This will be tracked by issue #1291. The reason why the diffuse analyses needs so many iterations needs also to be understood (see #1305).

Model	Unbinned	Binned	Stacked
Point	0.1 sec (4, 33105.355, 6.037e-16, -2.496)	7.3 sec (4, 17179.086, 5.993e-16, -2.492)	4.8 sec (4, 17179.081, 5.992e-16, -2.492)
Disk	18.9 sec (5, 34176.549, 83.633, 22.014, 0.201, 5.456e-16, -2.459)	127.9 sec (5, 18452.537, 83.633, 22.014, 0.202, 5.394e-16, -2.456)	<b>320.5 sec (30, 18451.250, 83.633, 22.015, 0.199, 5.395e-16, -2.462)</b>
Gauss	20.5 sec (4, 35059.430, 83.626, 22.014, 0.204, 5.374e-16, -2.469)	690.2 sec (4, 19327.126, 83.626, 22.014, 0.205, 5.346e-16, -2.468)	344.0 sec (4, 19327.270, 83.626, 22.014, 0.205, 5.368e-16, -2.473)
Shell	<b>118.9 sec (30, 35257.950, 83.632, 22.020, 0.281, 0.125, 5.844e-16, -2.449)</b>	<b>1509.9 sec (36, 19301.494, 83.633, 22.019, 0.289, 0.113, 5.787e-16, -2.447)</b>	<b>770.5 sec (34, 19301.186, 83.632, 22.021, 0.285, 0.118, 5.798e-16, -2.452)</b>
Ellipse	<b>151.9 sec (32, 35349.603, 83.626, 21.998, 44.725, 0.497, 1.994)</b>	<b>14291.3 sec (46, 19946.631, 83.627, 22.011, 44.980, 0.482, 1.940, 5.351e-16, -2.487)</b>	- not finished -
Diffuse	<b>1.4 sec (20, 32629.408, 5.299e-16, -2.672)</b>	<b>5792.3 sec (22, 18221.679, 5.688e-16, -2.657)</b>	<b>118.8 sec (25, 18221.814, 5.614e-16, -2.670)</b>

## #22 - 07/29/2014 04:32 PM - Knödseder Jürgen

- Status changed from In Progress to Closed

- % Done changed from 80 to 100

Close now since follow-up actions have been identified.

### Files

disk_unbin_scale3_DEC.png	38.7 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale3_RA.png	38.7 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale3_Radius.png	35 KB	07/25/2014	Knödseder Jürgen

disk_unbin_scale4_DEC.png	39.6 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale4_RA.png	39.6 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale4_Radius.png	39.5 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale5_DEC.png	53.1 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale5_RA.png	50.2 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale5_Radius.png	43.5 KB	07/25/2014	Knödseder Jürgen
disk_unbin_Npred_DEC.png	43.7 KB	07/25/2014	Knödseder Jürgen
disk_unbin_Npred_RA.png	46.7 KB	07/25/2014	Knödseder Jürgen
disk_unbin_Npred_Radius.png	41.9 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale5_DE_r2a2.png	42.7 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale5_RA_r2a2.png	50.5 KB	07/25/2014	Knödseder Jürgen
disk_unbin_scale5_Radius_r2a2.png	44.9 KB	07/25/2014	Knödseder Jürgen
disk_bin_scale5_Radius_r2a2.png	46 KB	07/25/2014	Knödseder Jürgen
disk_bin_scale5_RA_r2a2.png	51.5 KB	07/25/2014	Knödseder Jürgen
disk_bin_scale5_DEC_r2a2.png	42.9 KB	07/25/2014	Knödseder Jürgen
gauss_unbin_scale5_RA_r2a2.png	45.6 KB	07/25/2014	Knödseder Jürgen
gauss_unbin_scale5_RA_r5a5.png	37.6 KB	07/25/2014	Knödseder Jürgen
no-azimuth-integration.png	46.9 KB	07/26/2014	Knödseder Jürgen
integration-psf.png	13.5 KB	07/26/2014	Knödseder Jürgen
integration-model.png	12.4 KB	07/26/2014	Knödseder Jürgen
disk_unbin-model_scale5_RA_prec5_grad0002.png	49 KB	07/27/2014	Knödseder Jürgen
disk_unbin-model-switch_scale5_RA_prec5_grad0002.png	47.1 KB	07/27/2014	Knödseder Jürgen
gauss_5iter.png	47.5 KB	07/28/2014	Knödseder Jürgen
gauss_6iter.png	46.3 KB	07/28/2014	Knödseder Jürgen
gauss_7iter.png	46.9 KB	07/28/2014	Knödseder Jürgen
disk_7iter_prec5-diff.png	51 KB	07/28/2014	Knödseder Jürgen
disk_6iter_prec5-diff.png	41.7 KB	07/28/2014	Knödseder Jürgen
disk_5iter_prec5-diff.png	51.1 KB	07/28/2014	Knödseder Jürgen
disk_psf-switch_5iter_prec5-diff.png	42.8 KB	07/28/2014	Knödseder Jürgen
disk_psf-switch_8iter_prec5-diff.png	43.7 KB	07/28/2014	Knödseder Jürgen